Vertical shear instability in the Solar Nebula

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Introduction

The Vertical Shear Instability (VSI) is a potential route to purely hydrodynamic turbulence in protoplanetary disks (PPDs). However, VSI requires rapid cooling to be efficient. We quantify the thermodynamic dependence of the VSI using linear theory, and apply our results to evaluate the efficiency of the VSI in PPDs. We find the VSI can operate from $\sim$ 5AU to $\sim$ 50AU in a typical PPD, with characteristic growth times $\sim$ 30 orbits. The VSI is thus dynamically important in the outer disk, with significant implications for mass accretion, dust evolution, and thus planet formation.

Vertical shear and instability

- Astrophysical disks generally possess vertical shear, $\partial_z\Omega \neq 0$, because of baroclinicity, $\nabla P \times \nabla \rho \neq 0$.
- Example: vertically isothermal, radially non-isothermal disk,

$$r\frac{\partial\Omega}{\partial z} \simeq \frac{1}{2}\left(\frac{z}{H}\right) hq\Omega_{\mathrm{Kep}}.$$

- $h \equiv H/r \ll 1$: disk aspect-ratio.
- $q \equiv d\ln T/d\ln r$: radial temperature gradient.

Instability due to $\partial_z\Omega$?

- Consider an exchange of two fluid elements, separated by $(\Delta r, \Delta z)$ in space. Conservation of specific angular momentum imply a change in the specific kinetic energy by

$$\Delta E \sim \Delta r^2\left(\Omega^2 + \frac{\Delta z}{\Delta r}\cdot r\frac{\partial\Omega^2}{\partial z}\right). \quad (1)$$

- Vertical shear is weak, **BUT**

$$\Delta E < 0 \quad \text{is possible if} \quad |\Delta z| \gg |\Delta r|, \quad \Rightarrow \quad \textbf{INSTABILITY!}$$

- Radially-narrow, vertically-elongated disturbances.

Super-fast cooling is needed!

Eq. 1 assumes efficient cooling. **How fast does the disk need to cool?**

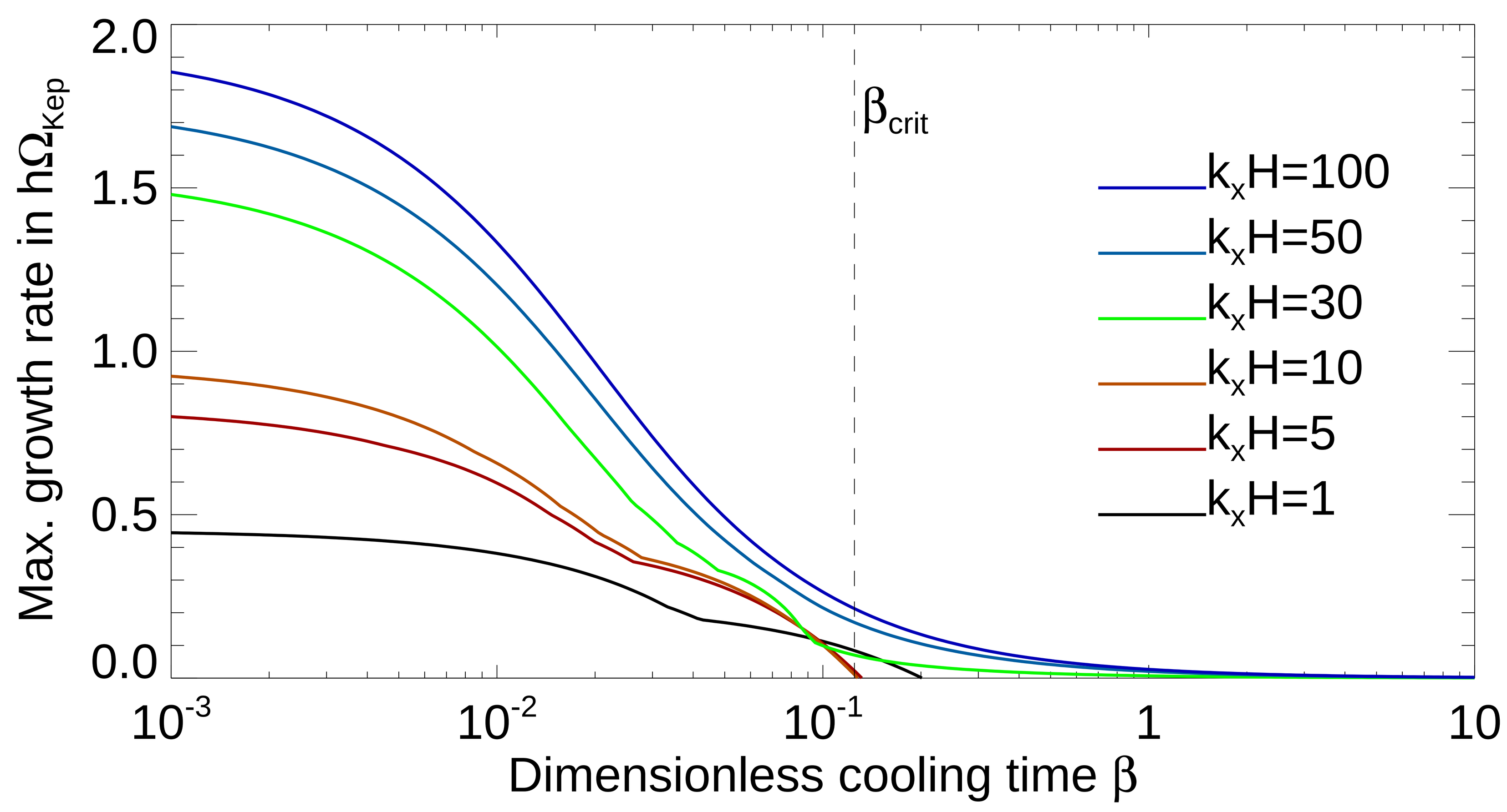
- Analytical condition:

$$t_{\mathrm{cool}}\Omega_{\mathrm{Kep}} < \frac{h|q|}{(\gamma-1)} \equiv \beta_{\mathrm{crit}} \ll 1.$$

- Fast cooling is required to overcome strongly stabilizing effect of vertical buoyancy (adiabatic index $\gamma > 1$).
- **Stringent thermodynamic requirement!**

Linear theory

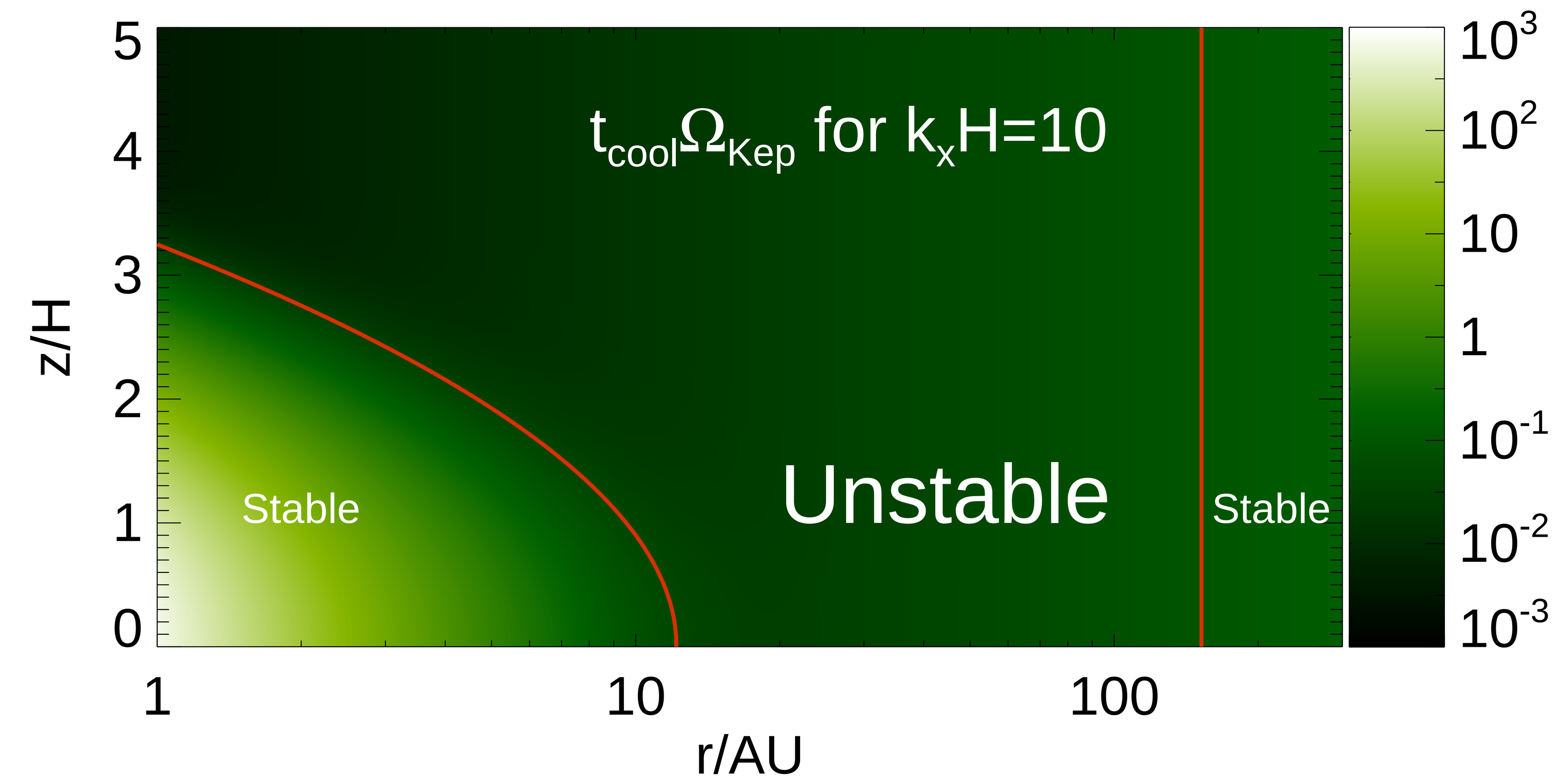
Growth rates from numerical solution to linearized fluid equations:



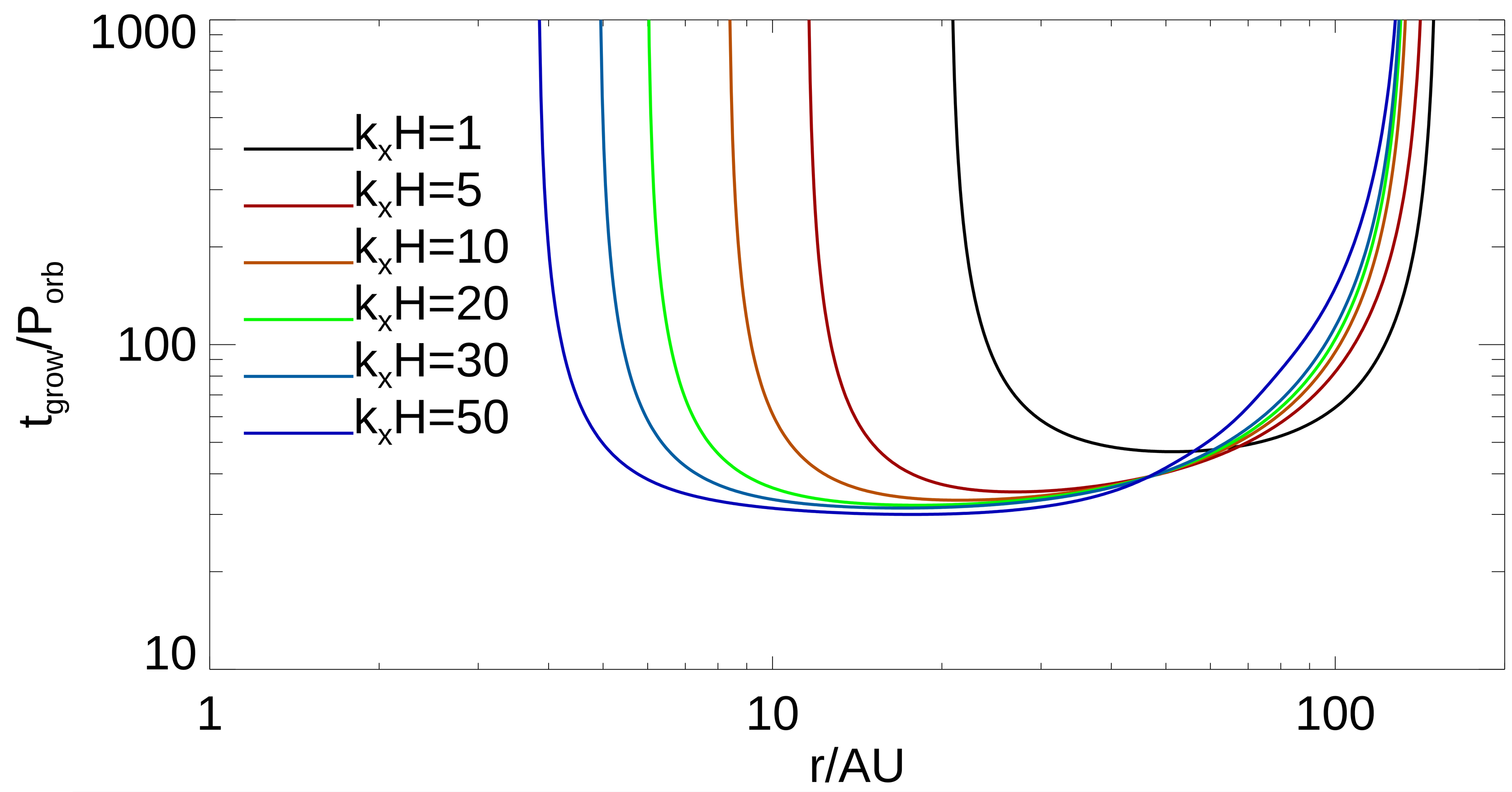
- $\beta = t_{\mathrm{cool}}\Omega_{\mathrm{Kep}}$; k_x: radial wavenumber.
- Rapid stabilization as cooling time is increased, even by small amounts.
- **$\beta < \beta_{\mathrm{crit}}$ is an excellent criteria for instability.**

VSI in the Minimum Mass Solar Nebula

Estimate actual cooling times in PPDs and compare with β_{crit}:

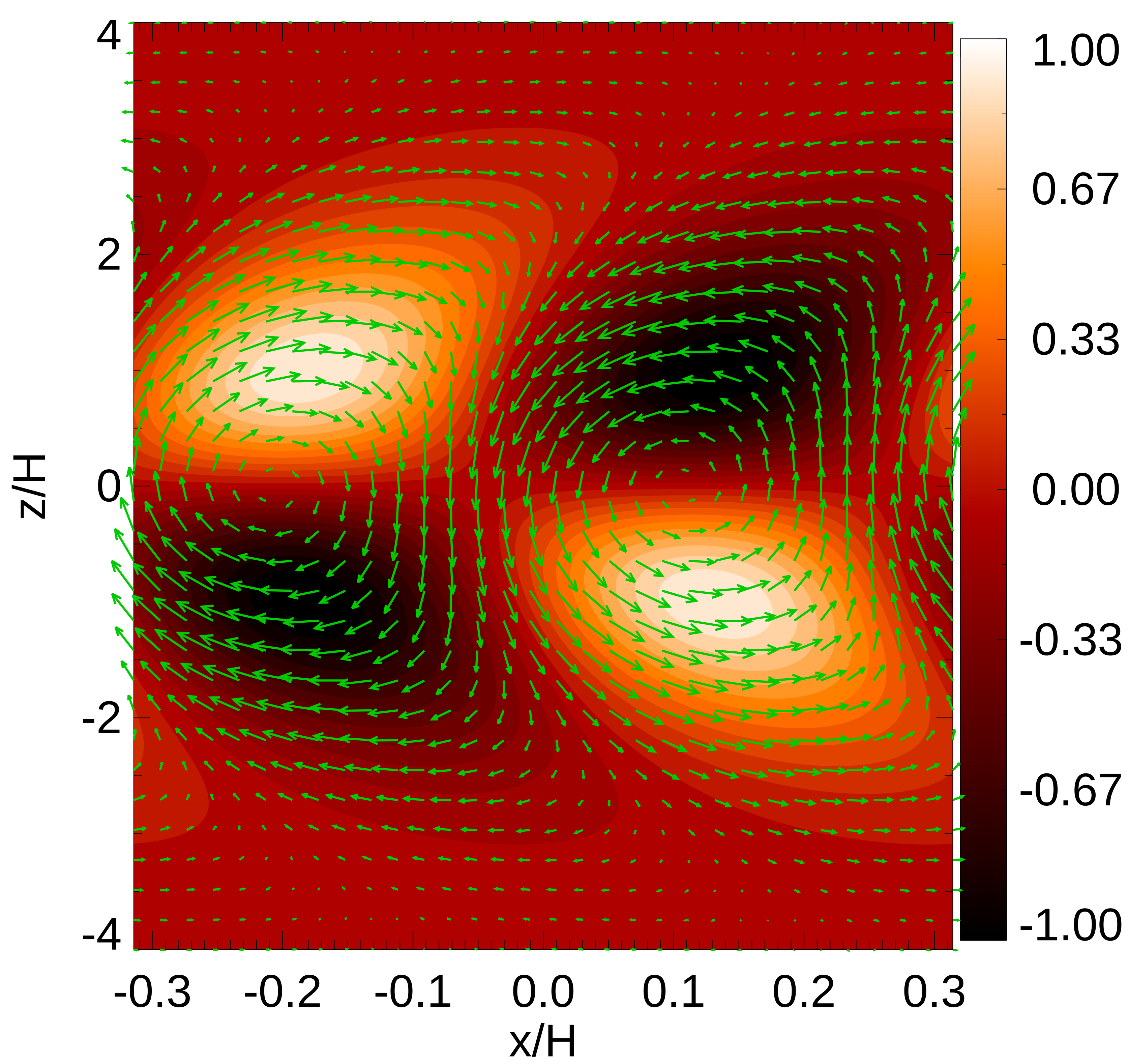


Numerical calculation of characteristic growth times:



- **Efficient VSI at tens of AU with growth times $\sim$ 30 orbits.**
- VSI occurs on very small scales towards the inner disk.

VSI in action



- $k_xH = 10$; $\beta = 0.1$; arrows: $\sqrt{\rho}(\delta v_x, \delta v_z)$; color: δP.

Summary

Astrophysical disks with vertical shear, $\partial_z\Omega \neq 0$, are dynamically unstable if they can cool rapidly. We have shown these conditions are satisfied, and thus the VSI can operate, in typical protoplanetary disks at tens of AU. The VSI thus provides a viable route to purely hydrodynamic turbulent transport of angular momentum and hence mass accretion in protoplanetary disks.