Graph Your Own Blackbody Spectra

on a single graph

label the axes and λ_{max}

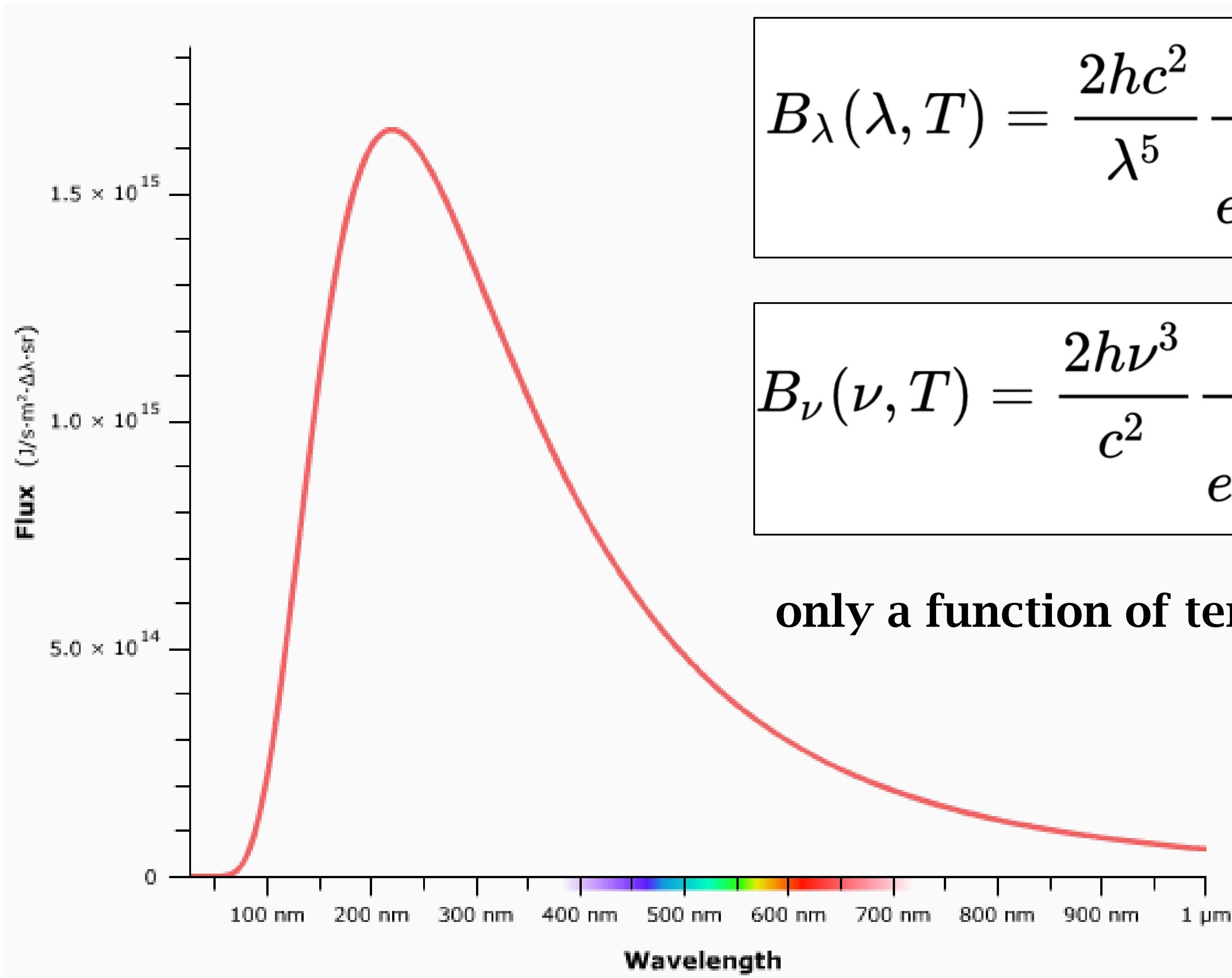
- T = 300 K
- T = 3,000 K
- T = 30,000 K

- $\lambda_{max} \approx (3 \times 10^6) / T$ nm

"Blackbody Spectrum"

Aka: thermal spectrum, Planck spectrum

characteristic of an "opaque" object



$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

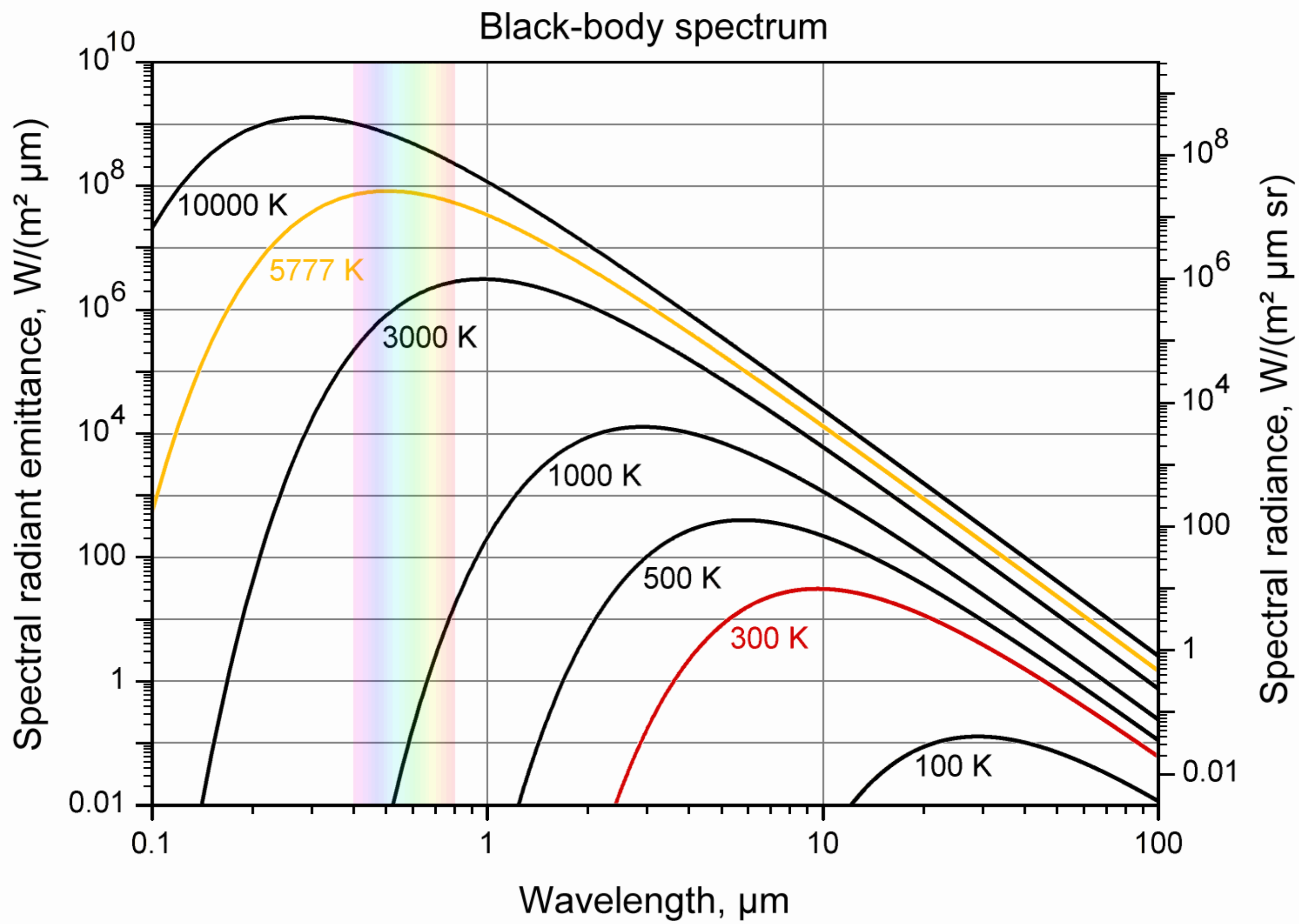
$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{k_B T}} - 1}$$

only a function of temperature!

Blackbody Spectra

The curves do not cross!

$B_\lambda(T)$



$B_\nu(T)$

How would you locate the peak wavelength (λ_{max}) of the Planck function?

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

Find the inflection point: $d(B_\lambda(T)) / d\lambda \equiv 0$

let $x \equiv hc/\lambda kT$

after differentiating:

$xe^x / (e^x - 1) = 5$

solution is $x = 4.965$

Can show that: $\lambda_{max} T = hc / 4.965\, k = 0.2898$ cm K

λ_{max} (nm) $\approx (3 \times 10^6) / T$ (Wien's Law)

Wien's Law

$$\lambda_{max} T = (2.898 \times 10^{-1}) \text{ cm K}$$

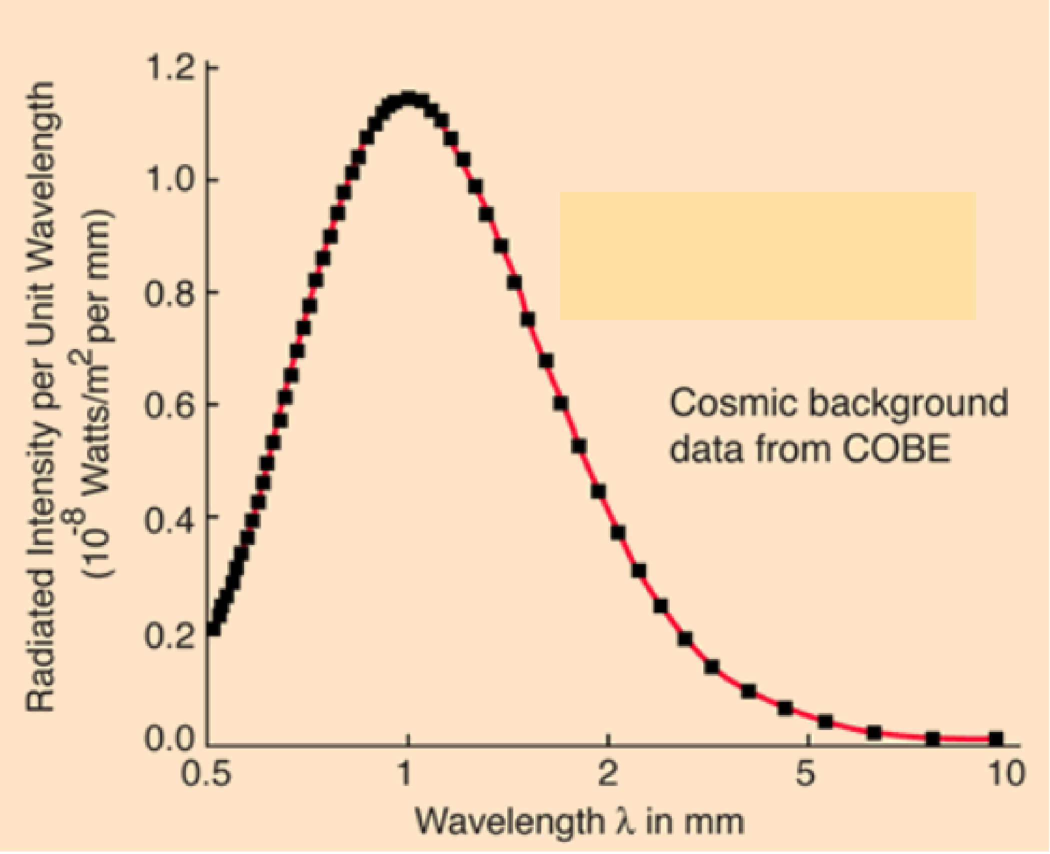
$$\nu_{max} / T = (5.879 \times 10^{10}) \text{ Hz K}^{-1}$$

I've memorized:

$$\lambda_{max} \approx (3 \times 10^{6}) / T \text{ nm}$$

What is the temperature of this light source?

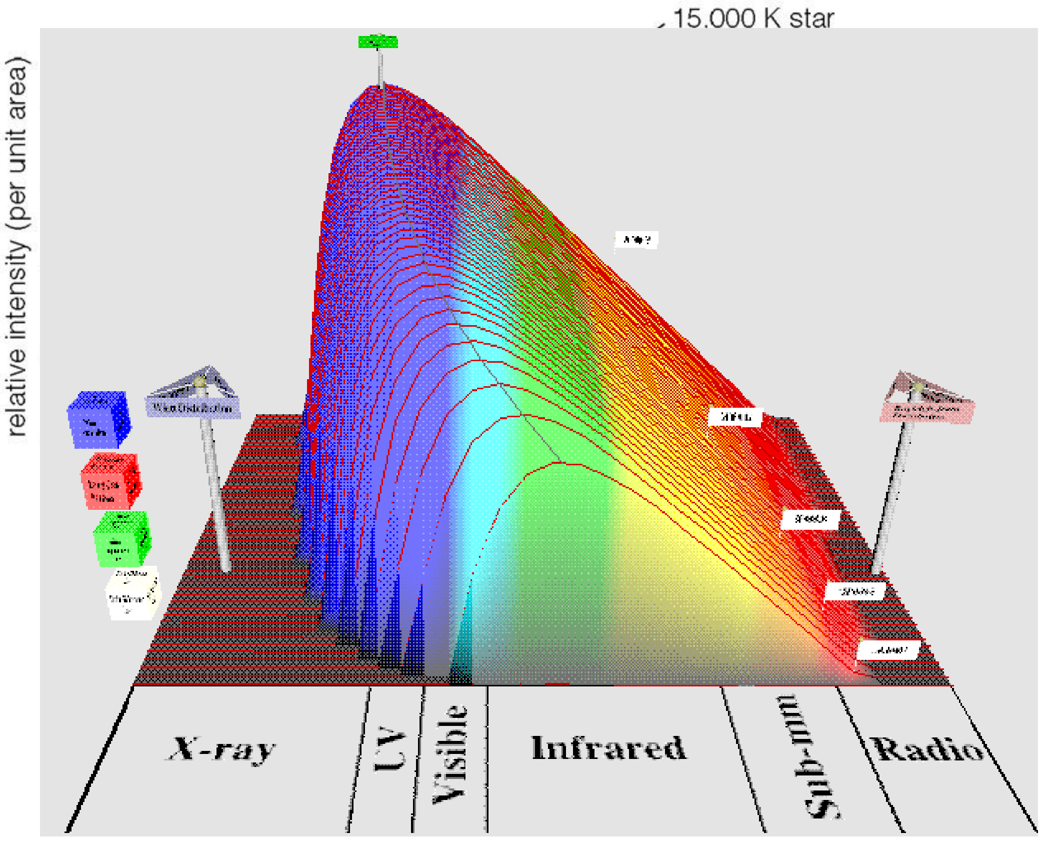
$$\lambda_{max} \approx \frac{3 \times 10^6}{T} \text{ nm}$$



A Blackbody "Spectrum"

specific shape, "continuous"

contains light of all wavelengths but not in equal amounts



Wien's "Law"

The hotter the object, the bluer its peak wavelength.

$$\lambda_{max} \approx \frac{3 \times 10^6}{T} \text{ nm}$$

Stefan-Boltzmann "Law"

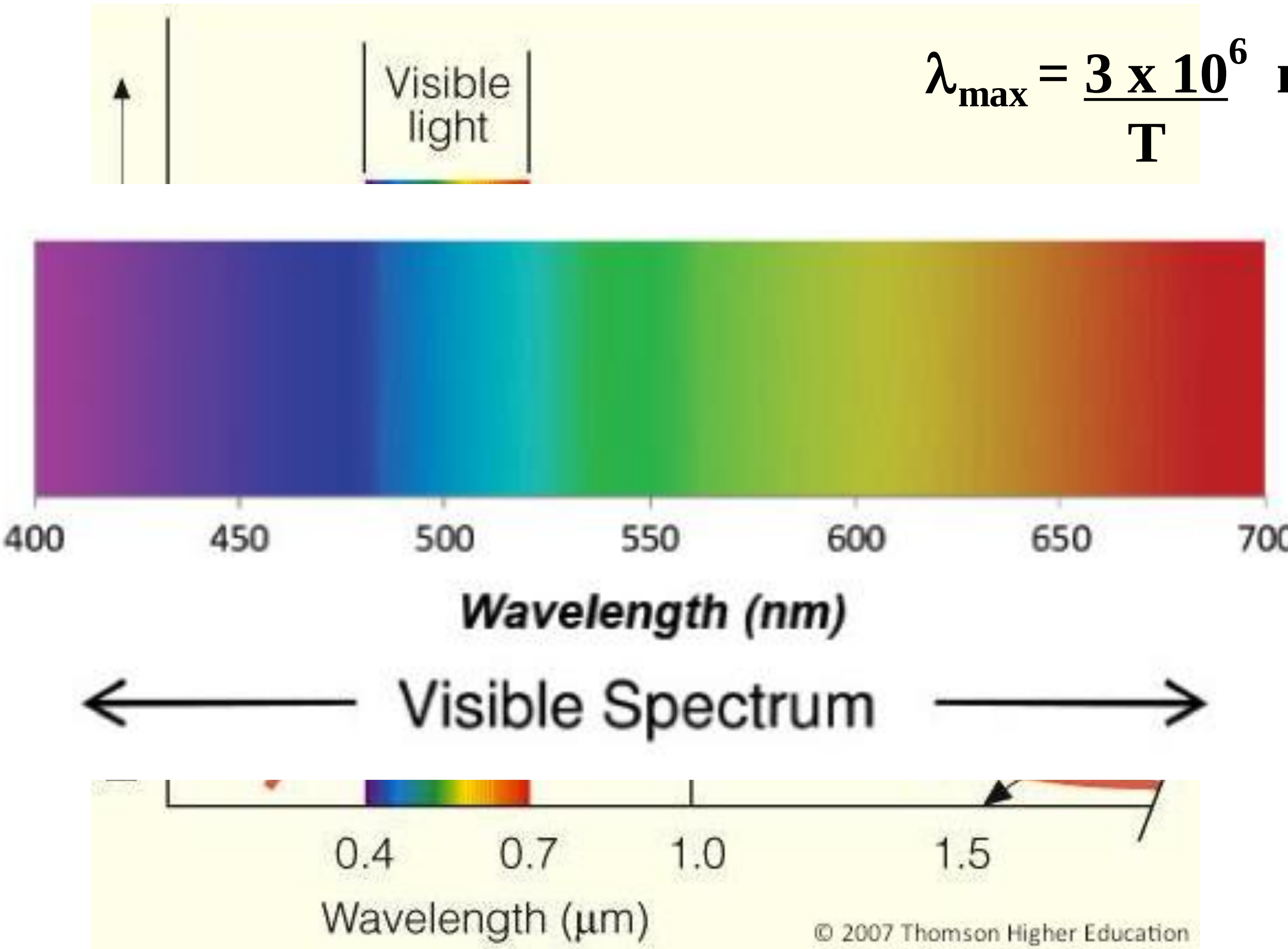
The total amount of light emitted from a given area depends on the temperature to the 4th power!!

total emitted energy / m^2 $= \sigma T^4$

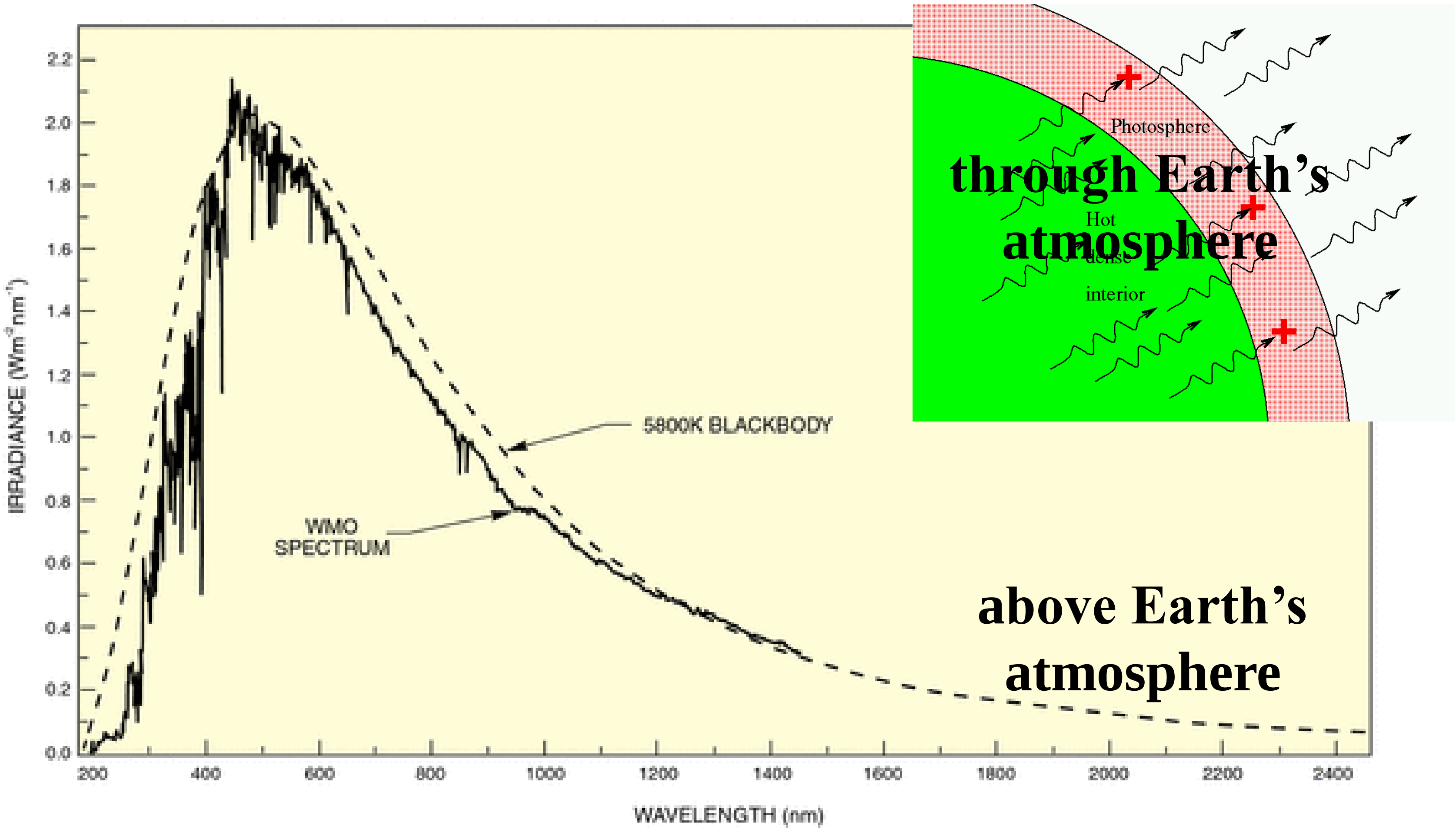
What color is the Sun?

T ~ 6000 K

$$\lambda_{max} = \frac{3 \times 10^6}{T} \text{ nm}$$



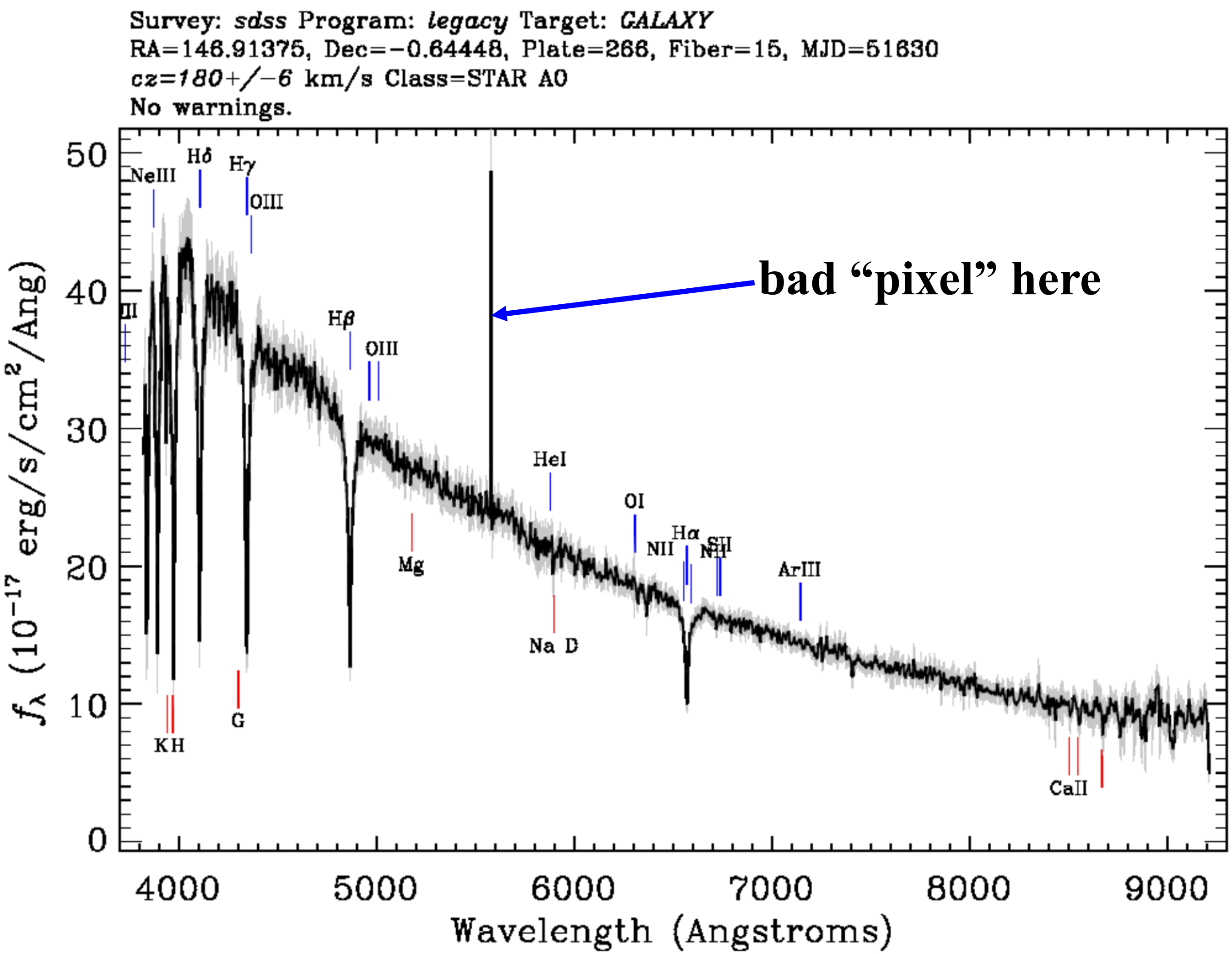
Spectrum of Our Sun
vs. 5800 K blackbody



through Earth's atmosphere

above Earth's atmosphere

Spectrum of an "A0" type Star
~10,000 K



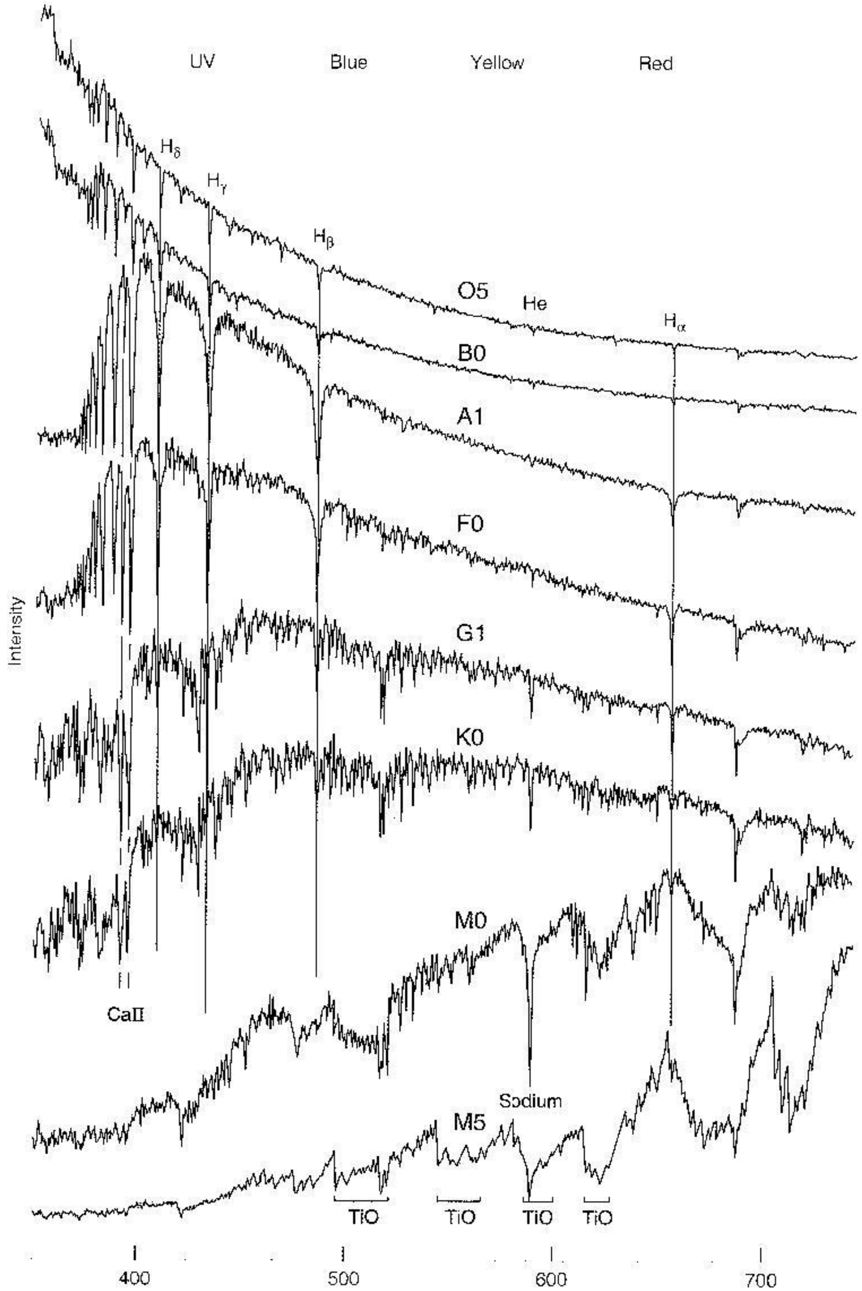
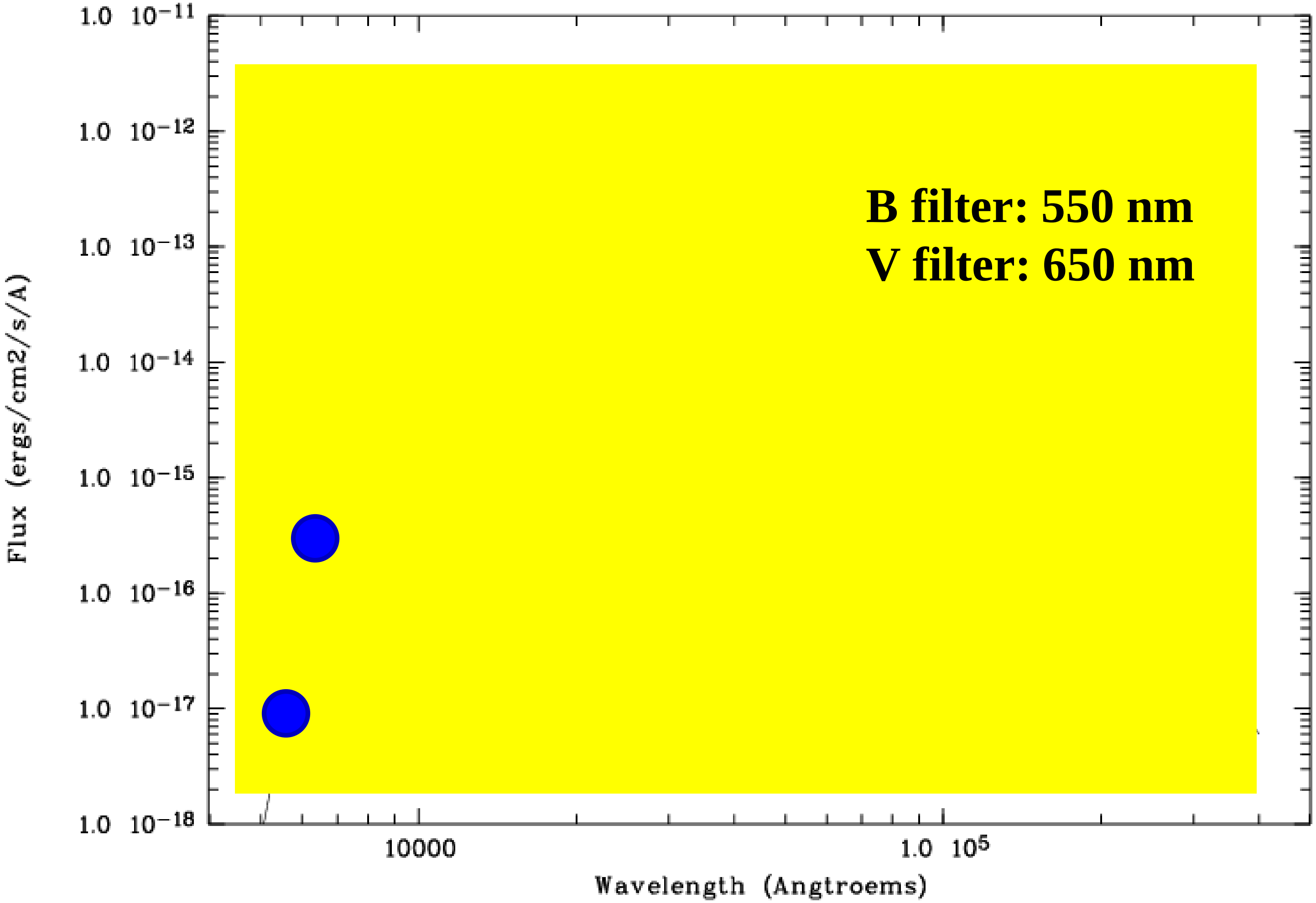


FIGURE 7-10
Modern digital spectra are often represented by graphs of intensity versus wavelength. Dark absorption lines are dips in intensity. The hottest stars are at the top and the coolest at the bottom. Hydrogen Balmer lines are strongest at about A0, while lines of ionized calcium (CaII) are strong in K stars. Titanium oxide (TiO) bands are strongest in the coolest stars. Compare these spectra with Figures 7-8c and 7-9. *(Courtesy NOAO, G. Jacoby, D. Hunter, and C. Christian)*

What is the approx. temperature of this star?

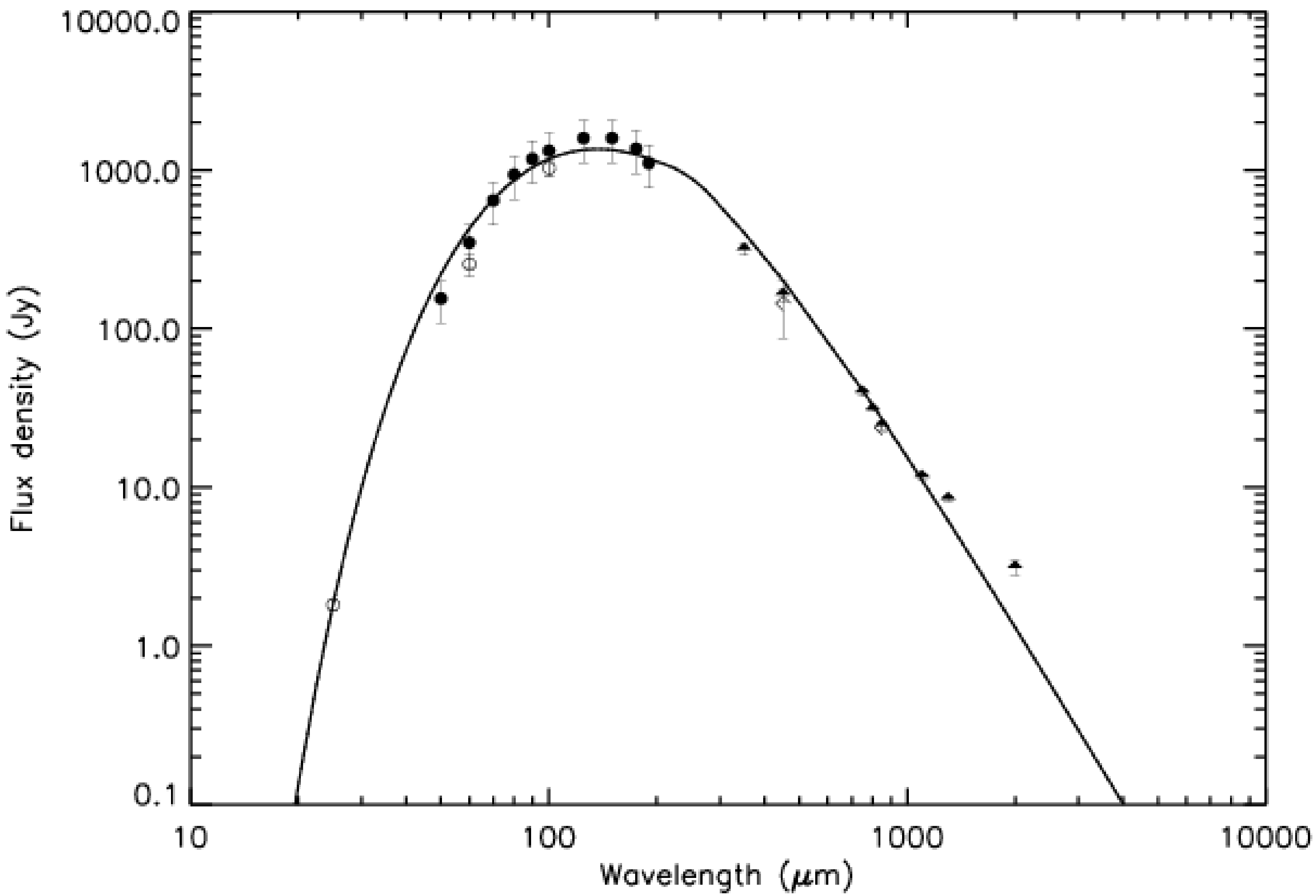
"color temperature"

B-V color index

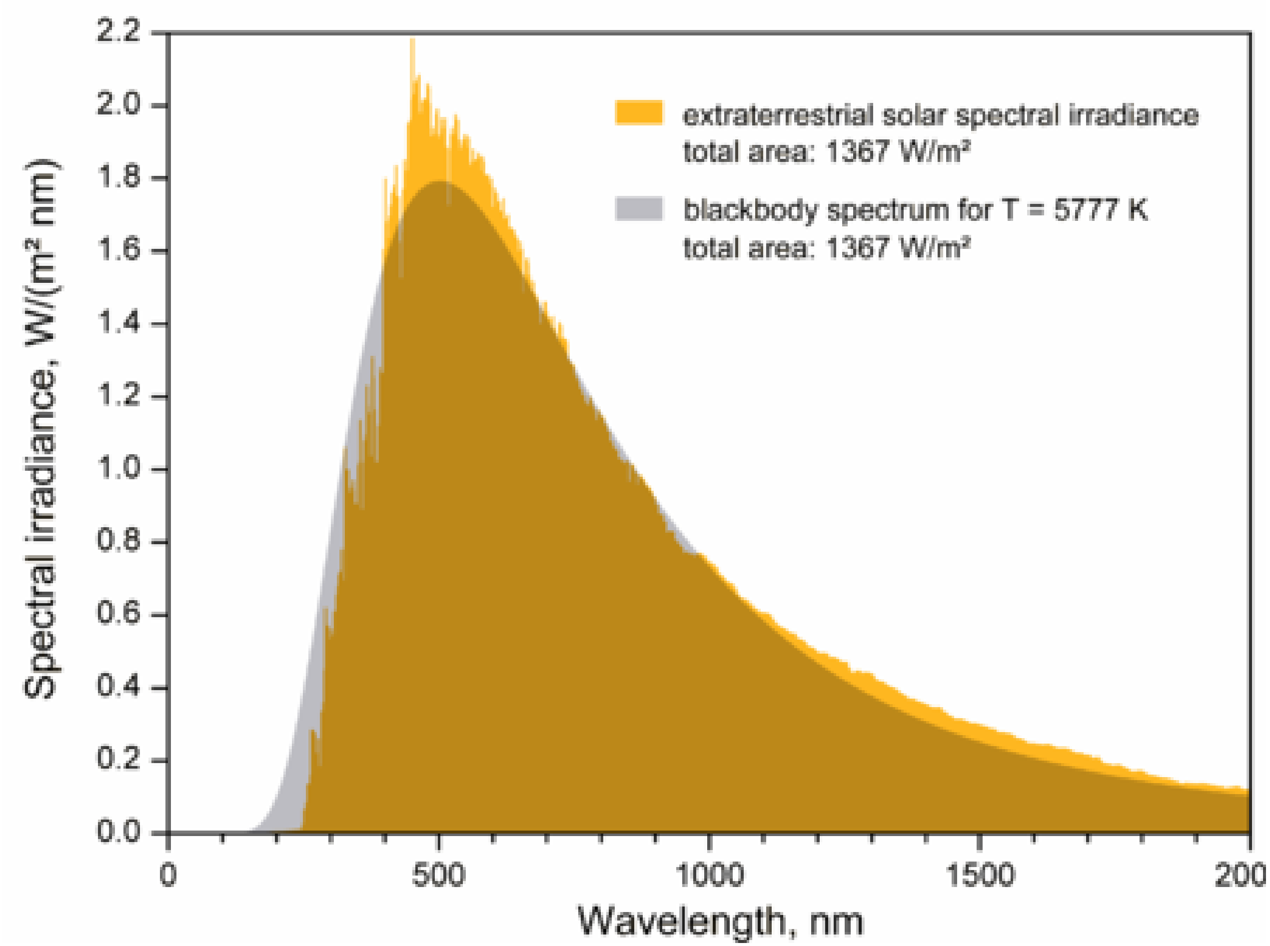


“Spectral Energy Distribution”

photometry through filters
approximation to a “spectrum”



“Effective Temperature”



The effective temperature of the Sun (5777 K) is the temperature a blackbody of the same size yielding the same total luminosity.

"Luminosity"

L = flux integrated over the surface area of the object

$$L = \int_{surface} F\, dA$$

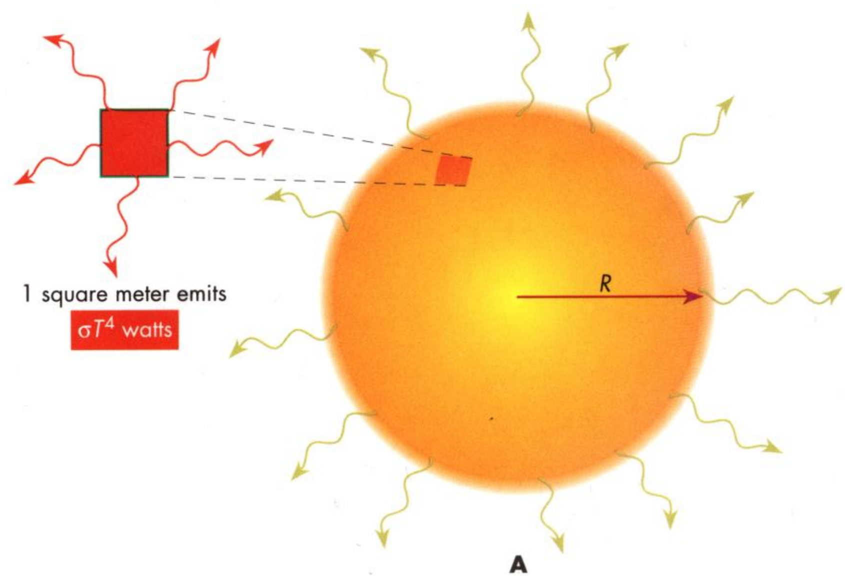
If F is constant and the surface is spherical

(radius = R), then

$$L = 4\pi R^2 \cdot F$$

where F is flux at the surface and

$4\pi R^2$ is surface area of the sphere



“Equilibrium Temperature”

- **What is the temperature of an object at a distance (D) from the Sun?**

- **Do you understand the problem?**
  - **What are some important factors?**
  - **Draw a diagram.**

- **Develop a plan.**

Energy Balance

In general, at a distance D from any star:
energy received = energy emitted

$$(1\text{-}A) \cdot [L_{star} / 4\pi D^2] \cdot \pi r^2 = 4\pi r^2 \sigma T^4$$

$$T^4 = (1\text{-}A) * (L_{star} / 4\pi D^2) / 4\sigma$$

$\sigma = 5.670367 \times 10^{-8}\ W \cdot m^{-2} \cdot K^{-4}$

r = radius of the absorbing object

A = albedo = reflectivity

energy received = energy emitted

$$(\text{absorb effic.}) * [L_{Sun} / 4\pi D^2] * \pi r^2 = 4\pi r^2 \sigma T^4$$

$$T^4 = (1\text{-}A) * \left(\frac{\text{solar constant}}{4\sigma}\right)$$

A = "albedo" = efficiency of reflection

Solution

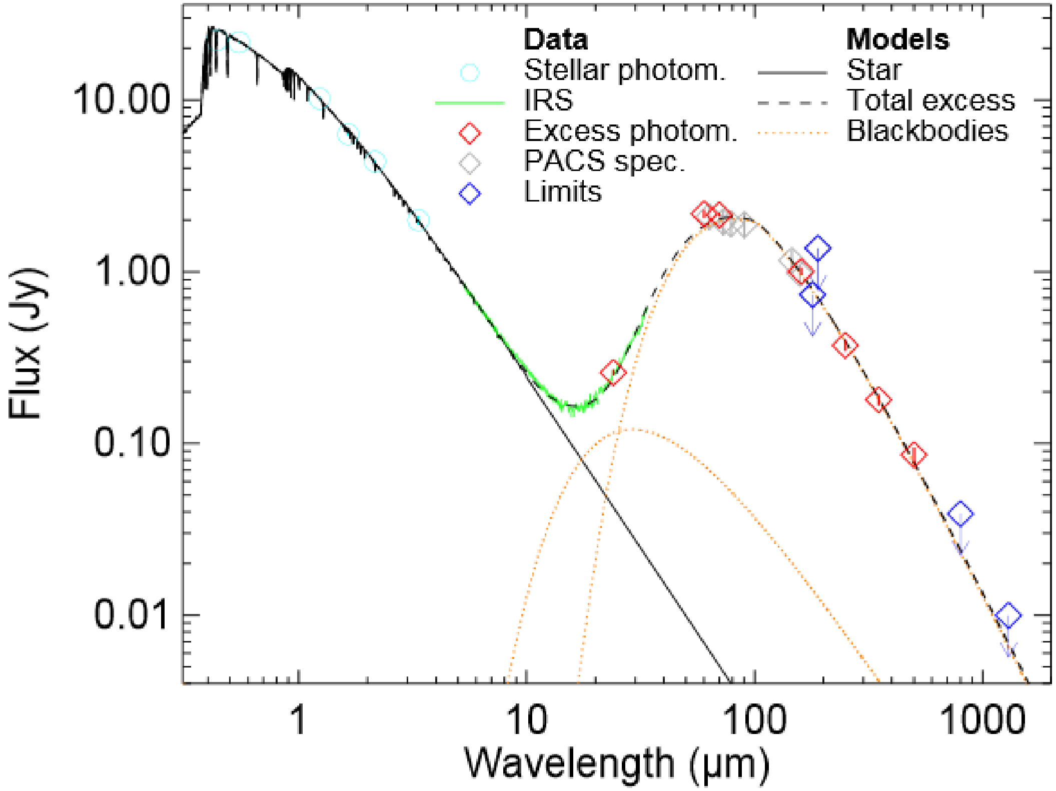
$$T^4 = (1-A) * (L_{star} / 4\pi D^2) / 4\sigma$$

$$D^2 = (1-A) * (L_{star}/16\pi\sigma T^4)$$

49 Ceti

A1V star with "infrared excess" => a "debris disk"

$L_{star} = 25\ L_{Sun}$



<u>How</u> would you plan to answer?

T_{eq} (Earth) ≈ 255 K

What are the temperatures?

How far from star is the debris in AU?

$$D^2 = (1\text{-}A) * (L_{star} / 16\pi\sigma T^4)$$

$$D \propto L^{1/2}\ T^{-2}$$

Special Regions in our Solar System & Exoplanets
“frost line” (~150 K); “habitable zone”; etc.