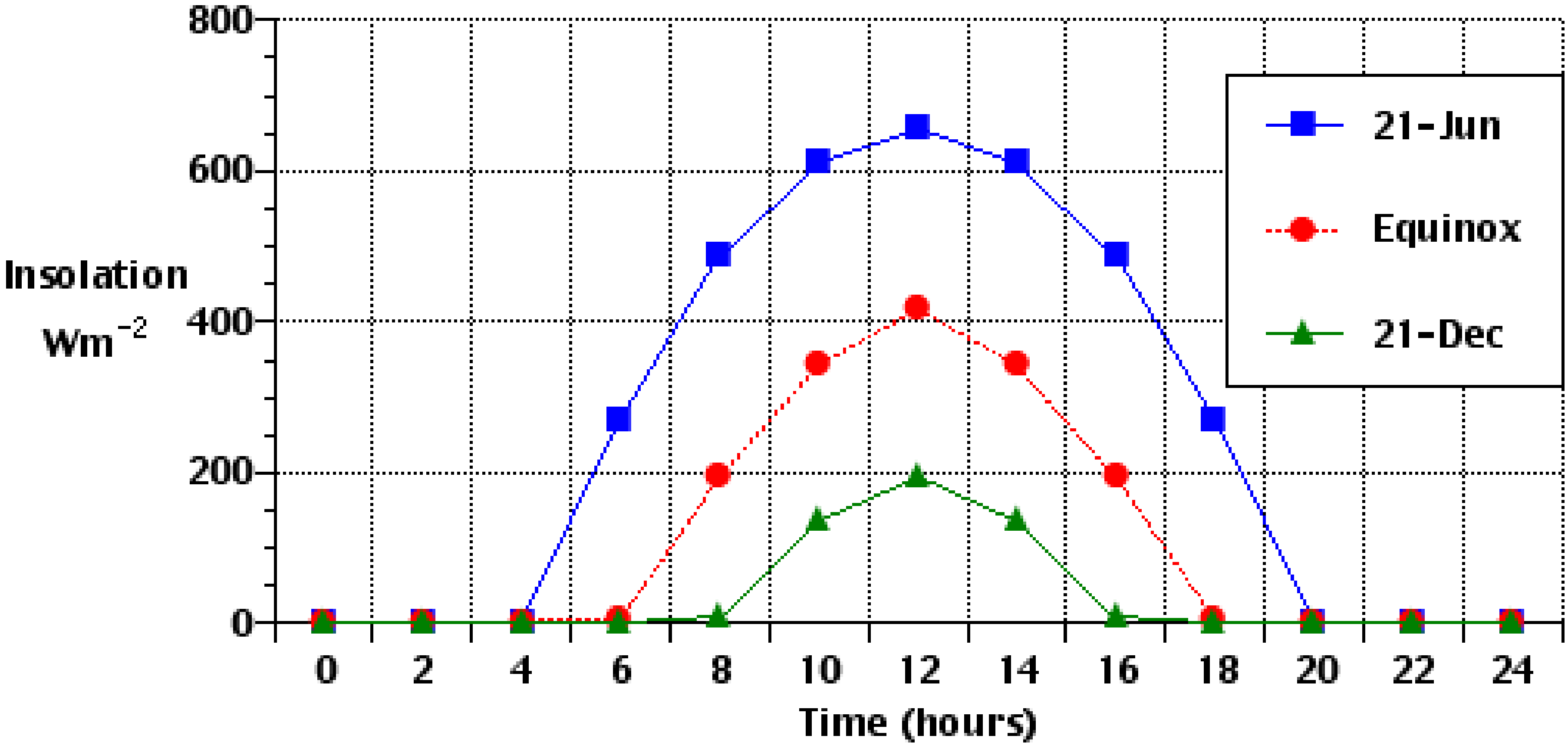
How many times more solar energy does Minneapolis receive during a day in summer day vs. winter?



Notice PLEASE

- Pick up today's Doodle Sheet
- Pick up past assignments in back
- Grades are posted

- Wednesday:
  - TIMESTEP

- Thursday:
  - Study session until 3 pm only
  - Colloquium ??

- Observing:
  - Venus, Moon, Mercury(!)
  - ISS orbital passes

Orbital Passes of the ISS

Date	Brightness (mag)	Start			Highest point			End		
		Time	Alt.	Az.	Time	Alt.	Az.	Time	Alt.	Az.
04 Feb	-1.5	19:10:50	10°	N	19:12:01	14°	NNE	19:12:01	14°	NNE
05 Feb	-1.3	19:58:52	10°	NW	19:59:53	18°	NW	19:59:53	18°	NW
06 Feb	-3.2	19:11:24	10°	NNW	19:14:34	38°	NE	19:14:58	36°	ENE
07 Feb	-2.4	18:24:08	10°	NNW	18:26:50	22°	NE	18:29:33	10°	E
07 Feb	-1.7	20:00:43	10°	WNW	20:03:19	26°	WSW	20:03:19	26°	WSW
08 Feb	-3.1	19:12:46	10°	NW	19:16:04	54°	SW	19:18:57	13°	SSE
09 Feb	-3.8	18:25:05	10°	NW	18:28:27	73°	NE	18:31:48	10°	SE
10 Feb	-0.7	19:15:20	10°	W	19:17:16	15°	SW	19:19:12	10°	SSW

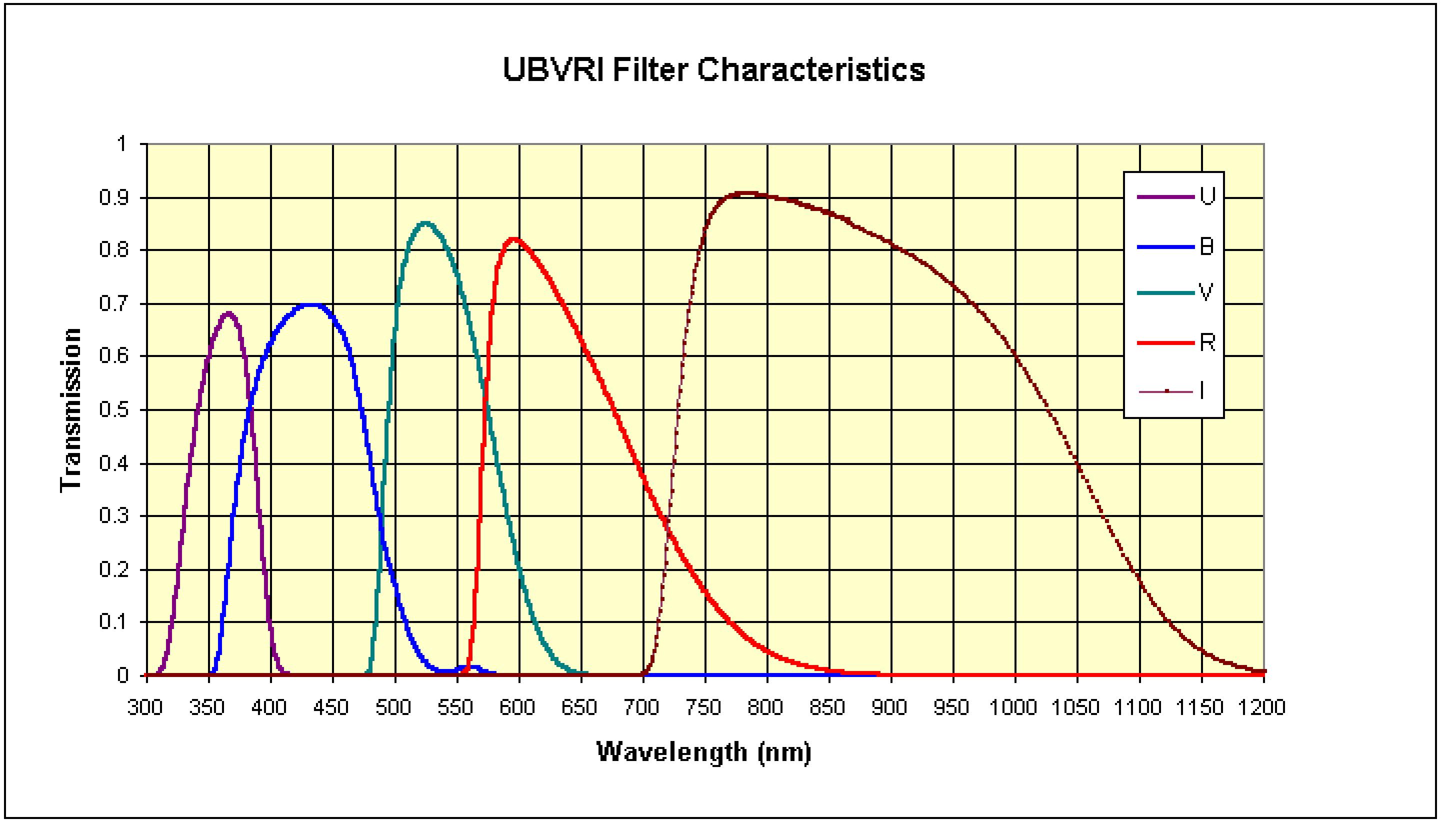
Review of Friday's Class
magnitudes and filters

- "flux" continued
- $\Delta m = (m_1 - m_2) = 2.5 \log (F_2/F_1)$
- apparent magnitude in visual band: m_v

- Today:
  - "absolute magnitudes": M_v, etc.
  - "zero magnitude": $m = -2.5 \log (F/F_{m=0})$
  - "distance modulus": $m - M = 5 \log (d) - 5$

Photometric Systems

- **Meaning of m_b, m_v, etc. (M_b, M_v, B, V, …)**



"Magnitudes"
very useful rules of thumb

- Every 5 magnitudes = ??
- Every 2.5 magnitudes = ??

- Every 1.0 magnitudes = ??
- Every 0.5 magnitudes = ??

- Homework:
  - Astronomers often use the approximation that a 1% change in brightness of a star corresponds to a change of 0.01 magnitudes. Justify this approximation.

Let one star have "zero magnitude"

magnitude difference = Δm = $(m_1 - m_2)$ = 2.5 log (F_2/F_1)

Let $m_1 \equiv 0$,
so $F_{m=0}$ is the flux from a zero-magnitude star
The star Vega is defined as zero-magnitude.

Then m_2 = -2.5 log $(F_2/F_{m=0})$

So, in general:

$$m = -2.5 \log (F/F_{m=0})$$

$m = m_v$ and $F = F_v$ are defined for some bandwidth, like visual ("v")

$$F_0(\lambda) = F_0(\nu) \cdot c/\lambda^2$$

flux for zero-magnitude

- **Here's an online calculator:**
  - https://www.gemini.edu/sciops/instruments/midir-resources/imaging-calibrations/fluxmagnitude-conversion

Table 1. UBVRIJHKLM Filters

Band	λ_{eff} μm	FWHM μm	$f_X(m_X = 0)$* Jy**	
U	0.365	0.066	1780	ultraviolet
B	0.445	0.094	4000	blue
V	0.551	0.088	3600	visible
R	0.658	0.138	3060	red
I	0.806	0.149	2420	
J	1.220	0.213	1570	↓ near-infrared ↓
H	1.630	0.307	1020	
K	2.190	0.390	636	
L	3.450	0.473	281	
M	4.750	0.460	154	

*$m_X = 0$ for a star with spectral type and luminosity A0 V.

**Note: 1 Jy = 10^{-26} W m^{-2} Hz^{-1}.

- f_X (m_X = 0) = zero point flux density of X. I.e., the flux density of a zero magnitude star (the star Vega)
- If the flux density of wavelength X is measured, then the mag is,

$$m = -2.5 \log \frac{f_x(\text{source})}{f_x(m_x = 0)}$$

Problem

Assume you have two stars with individual apparent magnitudes of 3 and 4. What is the apparent magnitude of the unresolved pair of stars?

[Since magnitudes are logarithmically related to flux, you cannot simply add two magnitudes together to find the total apparent magnitude of an unresolved binary star.]

Problem Solution
fluxes add – magnitudes do not

- $m_1 = -2.5 \log (F_1/F_{m=0}) = 3$
- $m_2 = -2.5 \log (F_2/F_{m=0}) = 4$
- $m_{total} = -2.5 \log [(F_1 + F_2) / F_{m=0}] = ?$

- $-0.4m_1 = \log (F_1/F_{m=0})$
- $F_1 = F_{m=0} * 10^{-0.4m1}$

- $m_{total} = -2.5 \log[10^{-0.4m1} + 10^{-0.4m2}]$
- $m_{total} = -2.5 \log (0.082)$
- $m_{total} = 2.64$

- Does this magnitude value make sense?

Absolute Magnitude

magnitude difference = Δm = $(m_1 - m_2) = 2.5 \log (F_2/F_1)$

$F = L/4\pi d^2$

$(m_1 - m_2) = 2.5 \log (L_2/d^2_2 \div L_1/d^2_1)$

Imagine the same star viewed at two distances:

- its real distance (d in parsecs) - with magnitude m
- an absolute distance (d_{10}) of 10 parsecs – with magnitude M

$(m-M) = 2.5 \log (d^2/d^2_{10})$

$m-M = 5 \log (d) - 5$

$m-M = -5 \log (p) -5$, where p = parallax angle in arcsec

m-M is called the *"distance modulus"*

Problem
What is the absolute visual magnitude of the Sun?

Absolute visual magnitude of Sun $= M_V = ?$

Apparent visual magnitude of Sun $= m_V = -26.7$

$m - M = 5 \log(d) - 5$

Problem Solution

- **Absolute visual magnitude of Sun = M_V = ?**
- **Apparent visual magnitude of Sun = m_V = -26.7**

- **$m_v - M_v = 5 \log (d) - 5$**
- **$M_v = m_v - 5 \log (d) + 5$**

- **$M_v = (-26.7) - 5 \log (1/206265 \text{ pc}) + 5$**
- **$M_v = 4.8$**

- **Does this answer make sense?**
- **Distance modulus = -26.7 – 4.8 = -31.5**

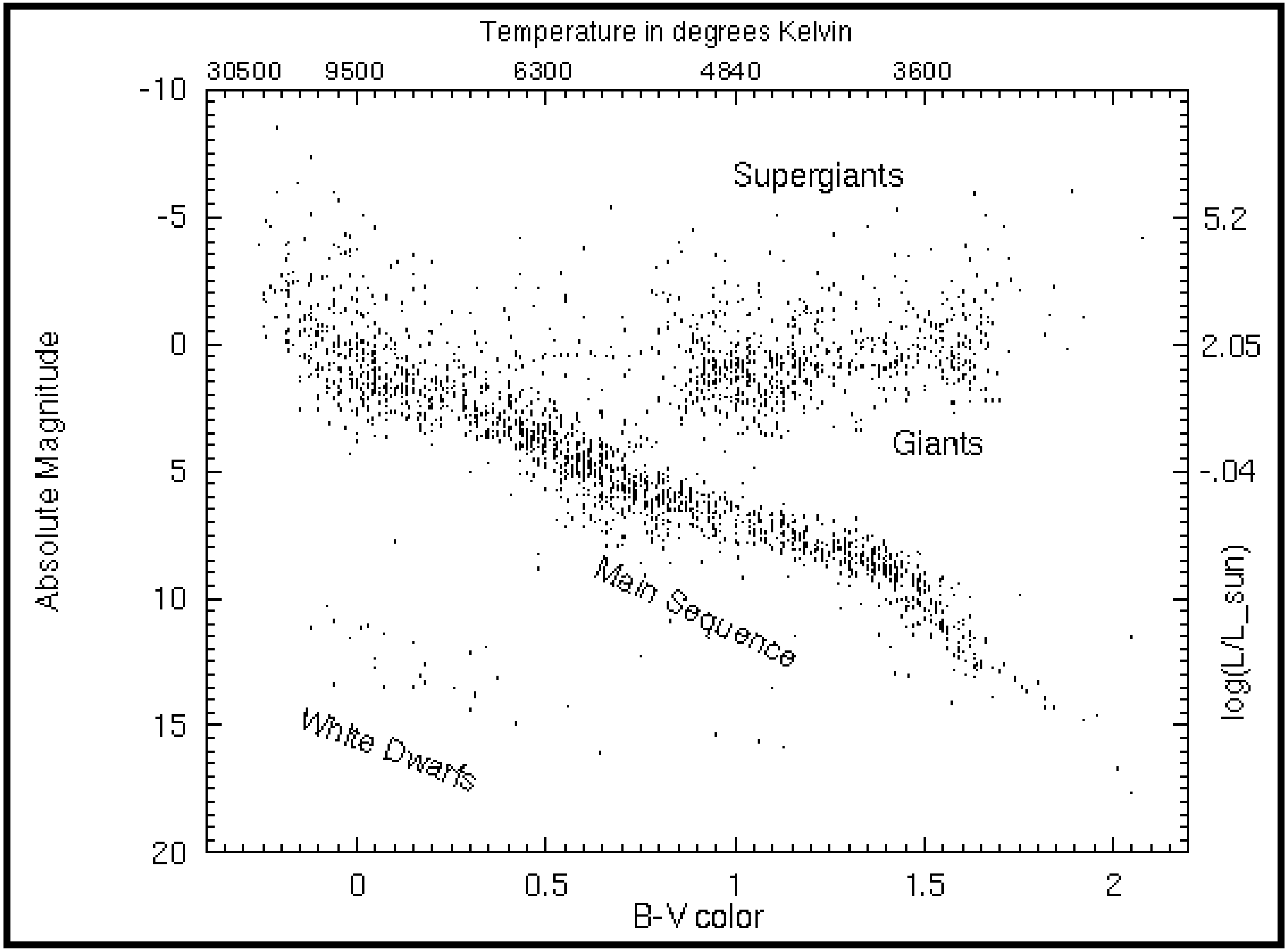
Another "Magnitude" Problem
"color index"

- **$m_U - m_B$**
  - **aka, U-B**

- **$m_B - m_V$**
  - **aka, B-V**

- **For an object with B-V > 0, is the object reddish or bluish?**

The Hertzsprung-Russell Diagram



"Blackbody Spectrum"

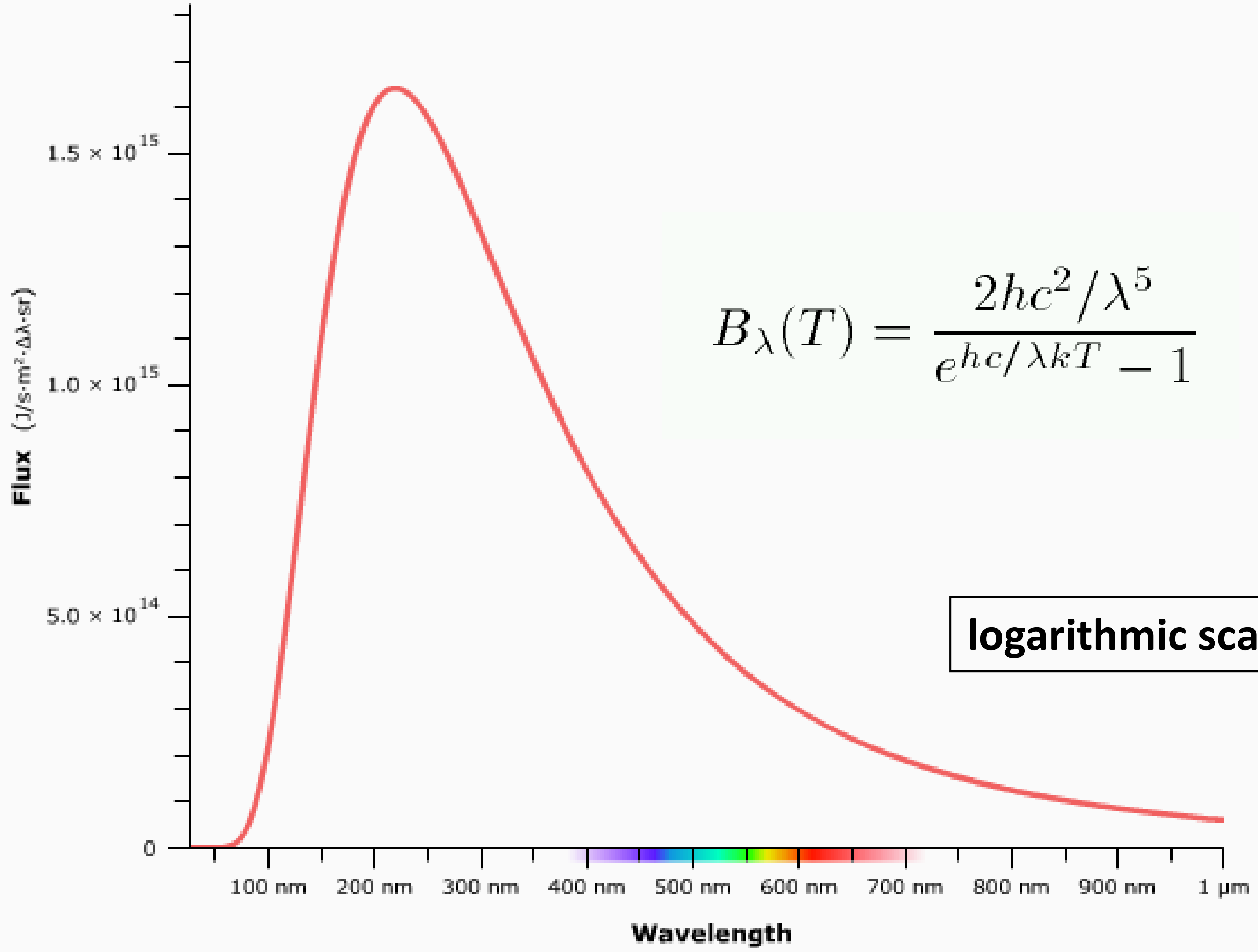
- **In your junior year Quantum Mechanics course, you will derive a formula for the flux of light emitted at different wavelengths (frequencies) from a perfectly absorbing body ("opaque").**
  - **the "Planck function" = $B_\lambda(T)$**
  - **"thermal radiation"**

- **The monochromatic flux emitted from the surface of such an object is**

- **$F_\lambda = \pi\, B_\lambda(T)$ erg sec^{-1} cm^{-2} Hz^{-1}**
- **$F_\nu = \pi\, B_\nu(T)$ erg sec^{-1} cm^{-2} cm^{-1}**

- **$\pi\, B_\lambda(T) = (\pi c/\lambda^2) \cdot B_\nu(T)$ because $d\lambda \neq d\nu$**

"Blackbody Spectrum"

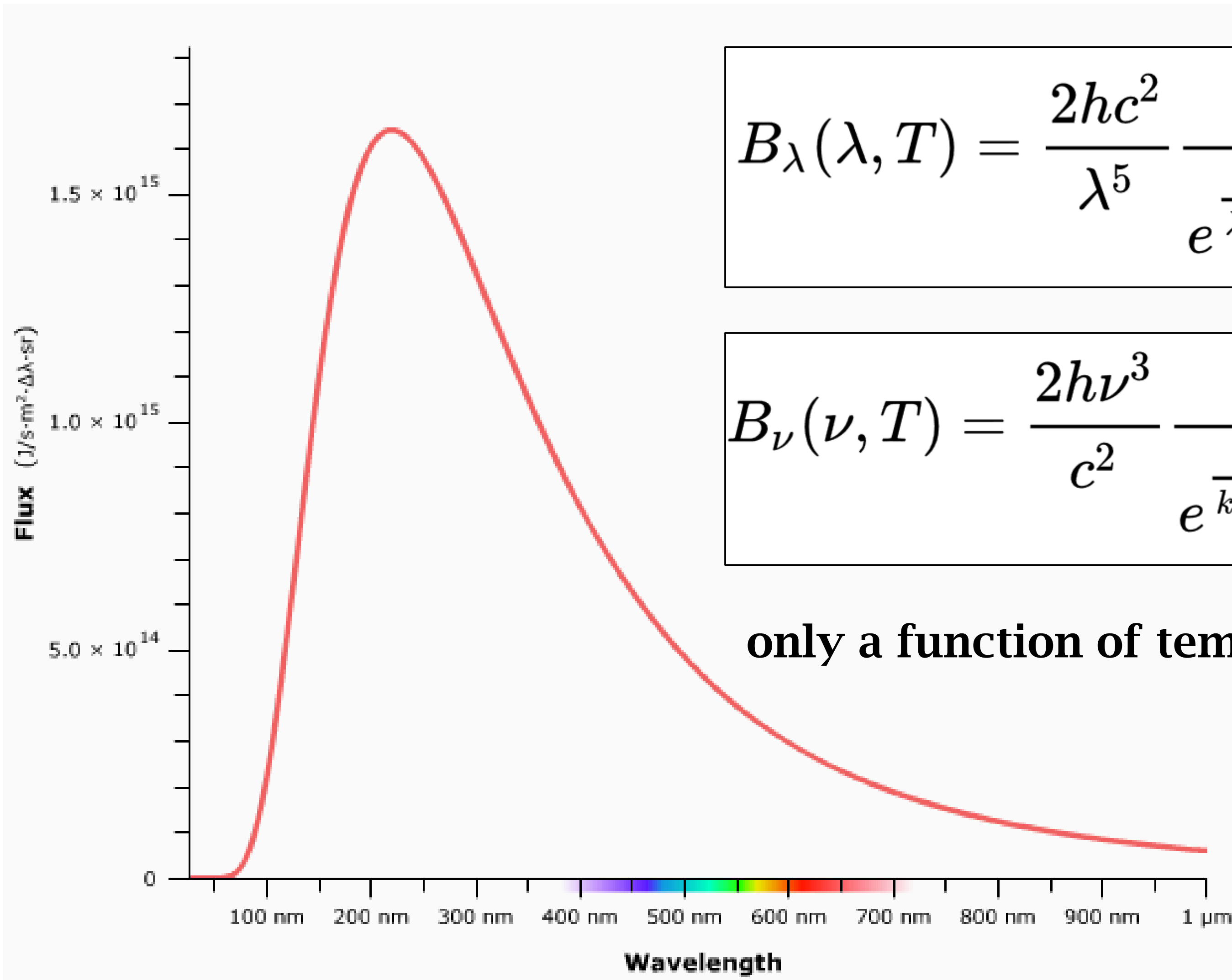


$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

logarithmic scale

What are the terms?

What's missing?



$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_\mathrm{B} T}} - 1}$$

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{k_\mathrm{B} T}} - 1}$$

only a function of temperature!

Approximations to the Planck Function

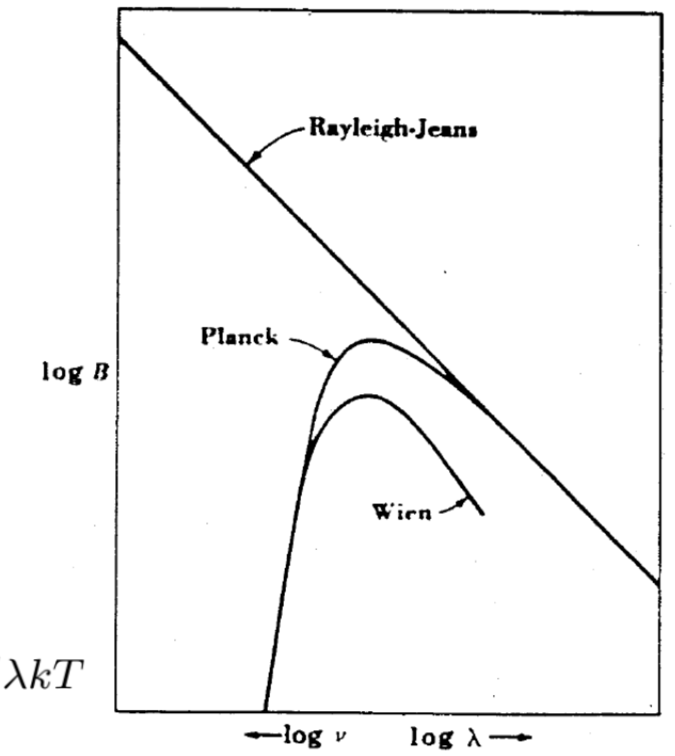
$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

- **For long wavelengths:**
  - **Rayleigh-Jeans distribution**
  - **For λ >> hc/kt**

$$B_\lambda(T) = \frac{2ck_{\mathrm{B}}T}{\lambda^4}$$

- **For short wavelengths:**
  - **Wien distribution**
  - **For λ << hc/kt**

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} e^{-hc/\lambda kT}$$



Stefan-Boltzmann Law
total emitted flux over all wavelengths

- **Integrate over wavelength (frequency):**

$$F = \pi \int_0^\infty B_\lambda(T)\, d\lambda$$

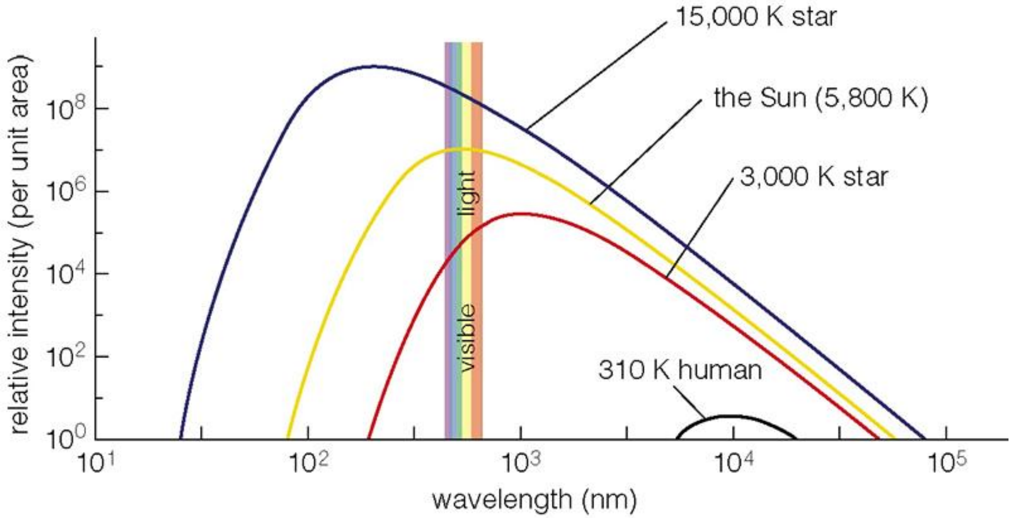
$$F = \sigma T^4$$

- **So, the luminosity of a star is:**
  - **$L = 4\pi R^2 F_{obs} = 4\pi R^2 \sigma T^4$ erg sec^{-1}**
  - **Stellar Luminosity α (Radius)^2 · (Temperature)^4**

A Continuous "Spectrum"

A spectrum is like a recipe!

contains light of all wavelengths but not in equal amounts



<u>Wien's "Law"</u>

The hotter the object, the bluer its color.

$$\lambda_{max} = \frac{3 \times 10^6}{T} \text{ nm}$$

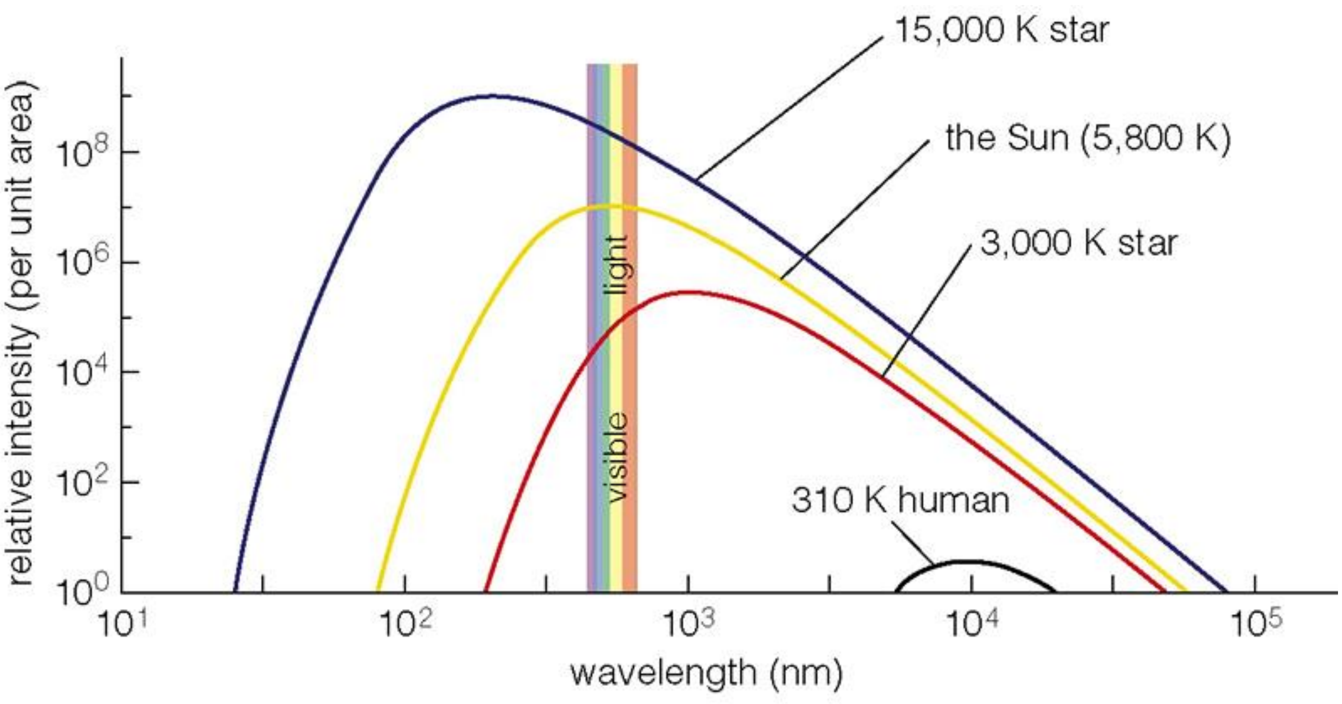
<u>Stefan-Boltzmann "Law"</u>

The total amount of light emitted from a given area depends on the temperature to the 4th power!!

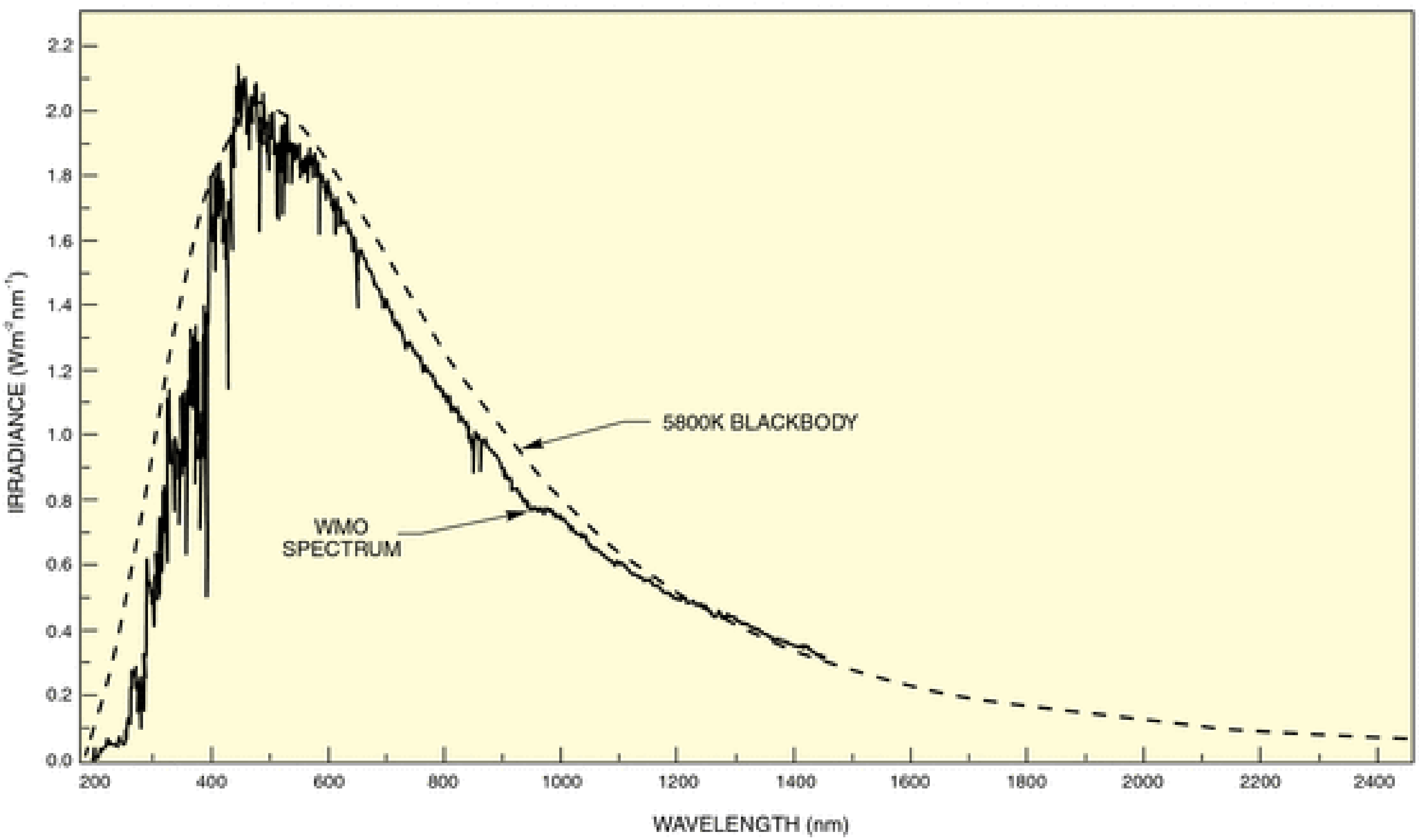
(Total Emitted Energy) α T^4

Emphasize a consequence of the Stefan-Boltzmann Law

Curves of different temperature never cross each other.



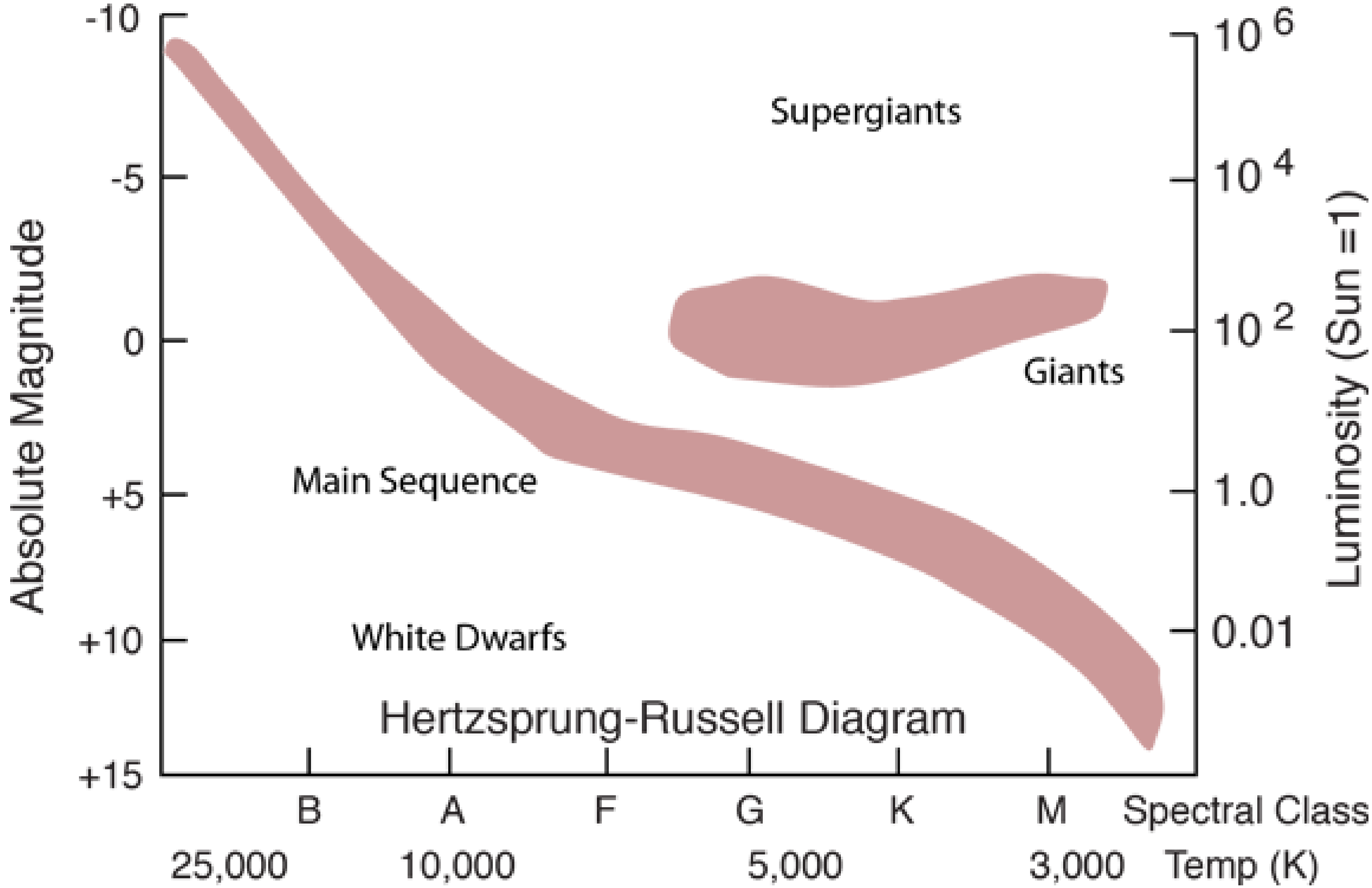
Spectrum of Our Sun
with and without Earth's atmosphere



The Hertzsrpung-Russell Diagram

What are the ranges of T and L?

Are those consistent?



How would you locate the peak wavelength (λ_{max}) of the Planck function?

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

Find the inflection point: $d(B_\lambda(T)) / d\lambda \equiv 0$

let $x \equiv hc/\lambda kT$

after differentiating:

$xe^x / (e^x - 1) = 5$

solution is $x = 4.965$

Can show that: $\lambda_{max} T = hc / 4.965\,k = 0.2898$ cm K

λ_{max} (nm) $\approx (3 \times 10^6) / T$ (Wien's Law)

Wien's Law

- $\lambda_{max}\, T \quad = (2.898 \times 10^{-1})$ cm K
- $\nu_{max} / T \quad = (5.879 \times 10^{10})$ Hz K^{-1}

- I've memorized:

$$\lambda_{max} \approx (3 \times 10^{6}) / T \quad \text{nm}$$

Graph Your Own Blackbody Spectra

label the axes and λ_{max}

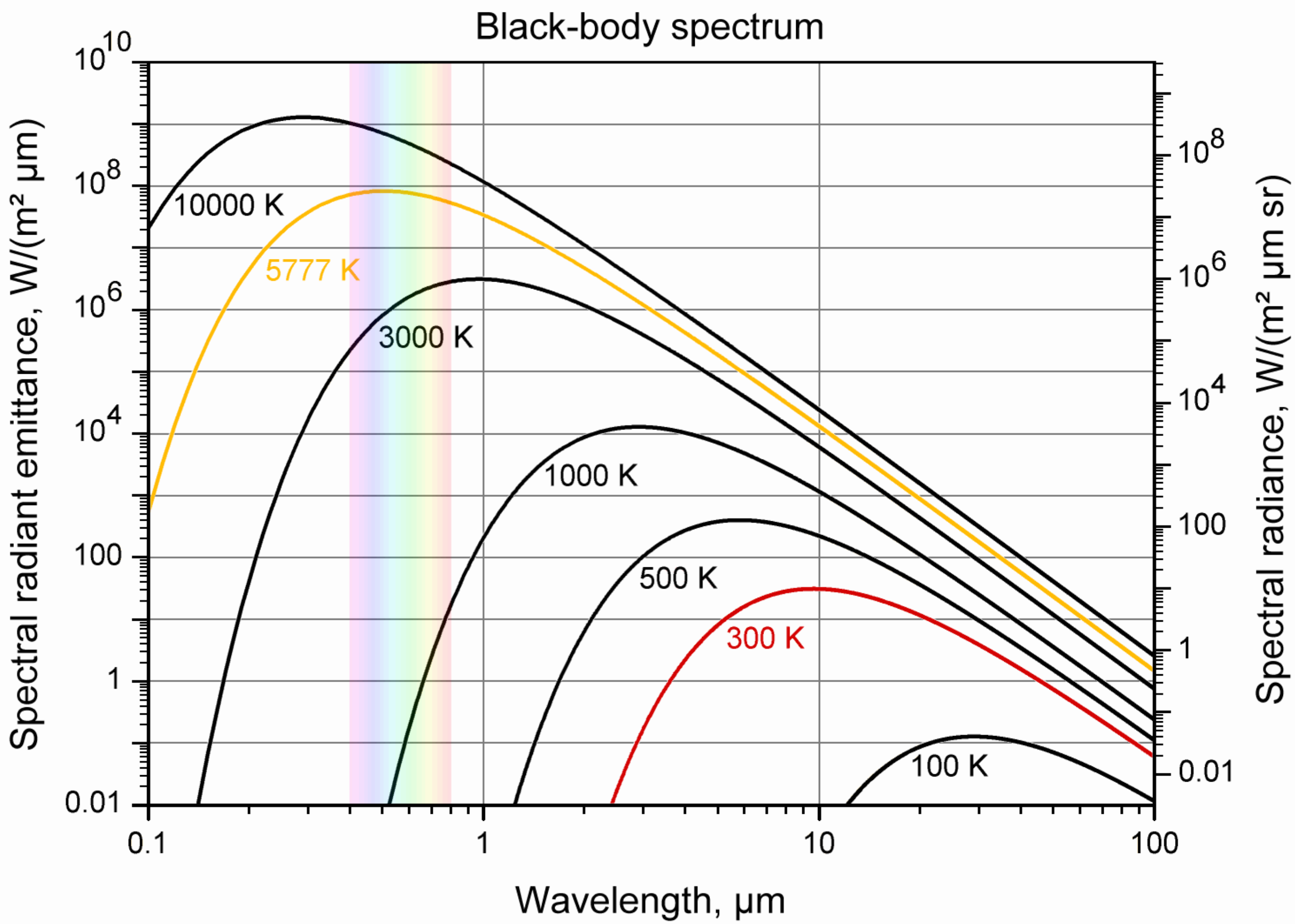
- **T = 300 K**
- **T = 3,000 K**
- **T = 30,000 K**

- $\lambda_{max} \approx (3 \times 10^6) / T$ **nm**

Blackbody Spectra

The curves do not cross!

$B_\lambda(T)$



$B_\nu(T)$