"How to Solve It"

Polya, 1945

Experts in different fields share a common problem-solving approach.

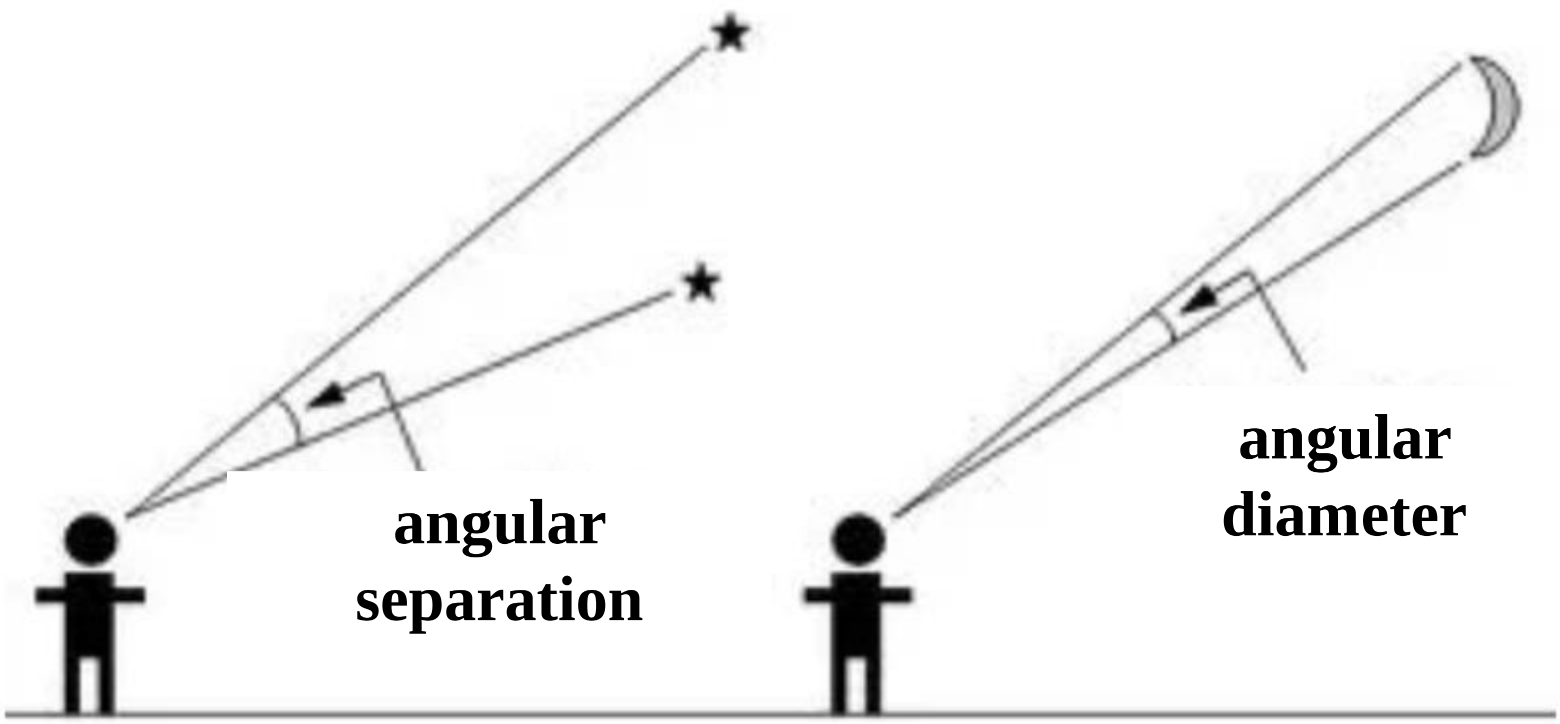
Understand the problem.

Devise a plan.

Carry out the plan and check each step along the way.

Look back and examine the solution.

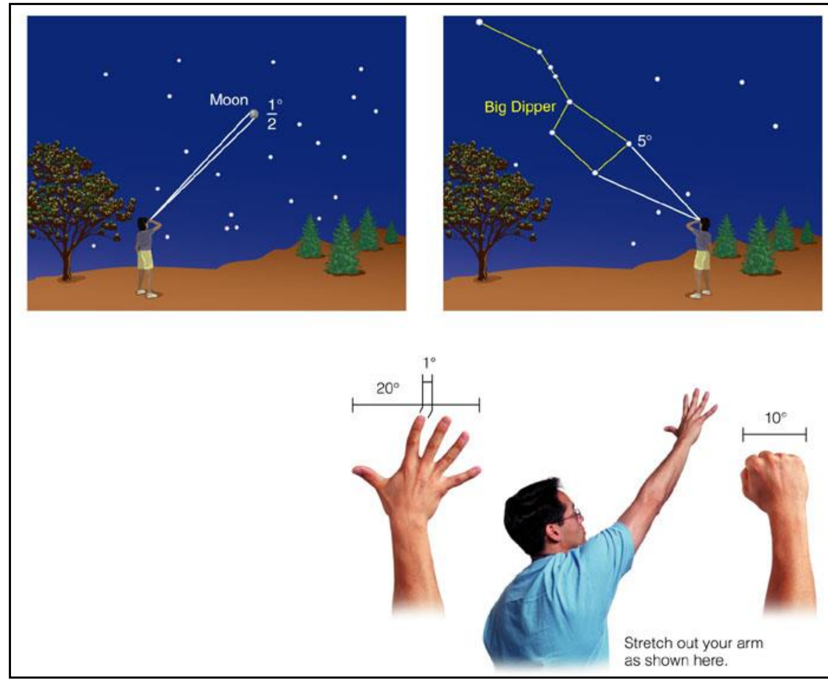
Angles and Terminology

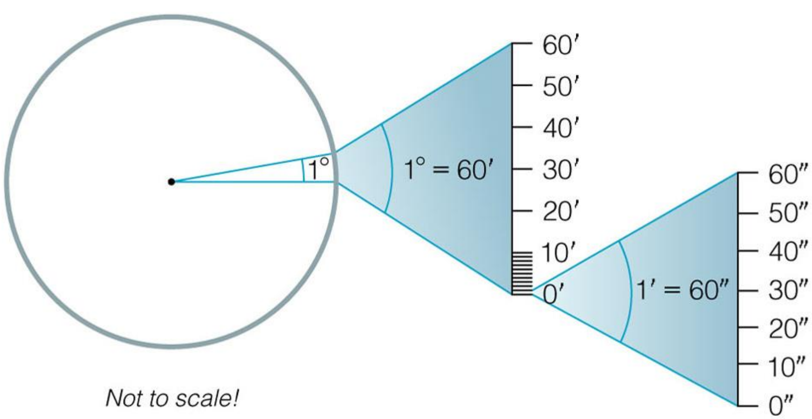


What is an Angle?

An angle is the "arc" between two different directions.

An object has an APPARENT size - an "angular size"





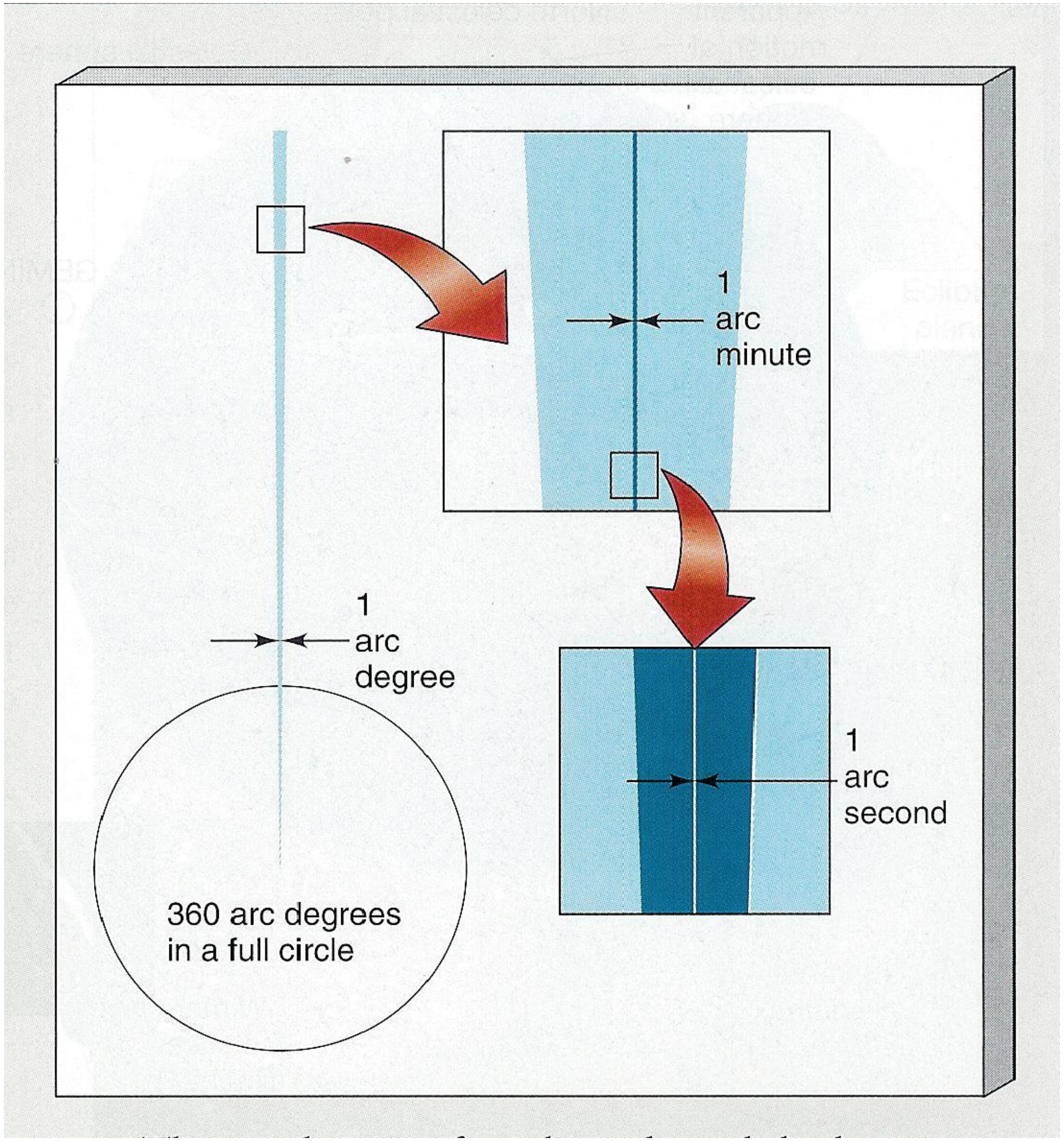
Each circle has 360 degrees = 360°

One degree = 60 arcminutes = 60'

The Sun & Moon appear to be 0.5° = 30 arcminutes

<u>How</u> convert between time and angle?

- **Earth rotates 360 degrees in 24 hours**
- **Do the math:**
  - **360 deg / 24 hr = 15 deg / hr**
- **<u>Problem</u>: In what amount of time would the Sun appear to move its own angular diameter?**



one "arcsecond"

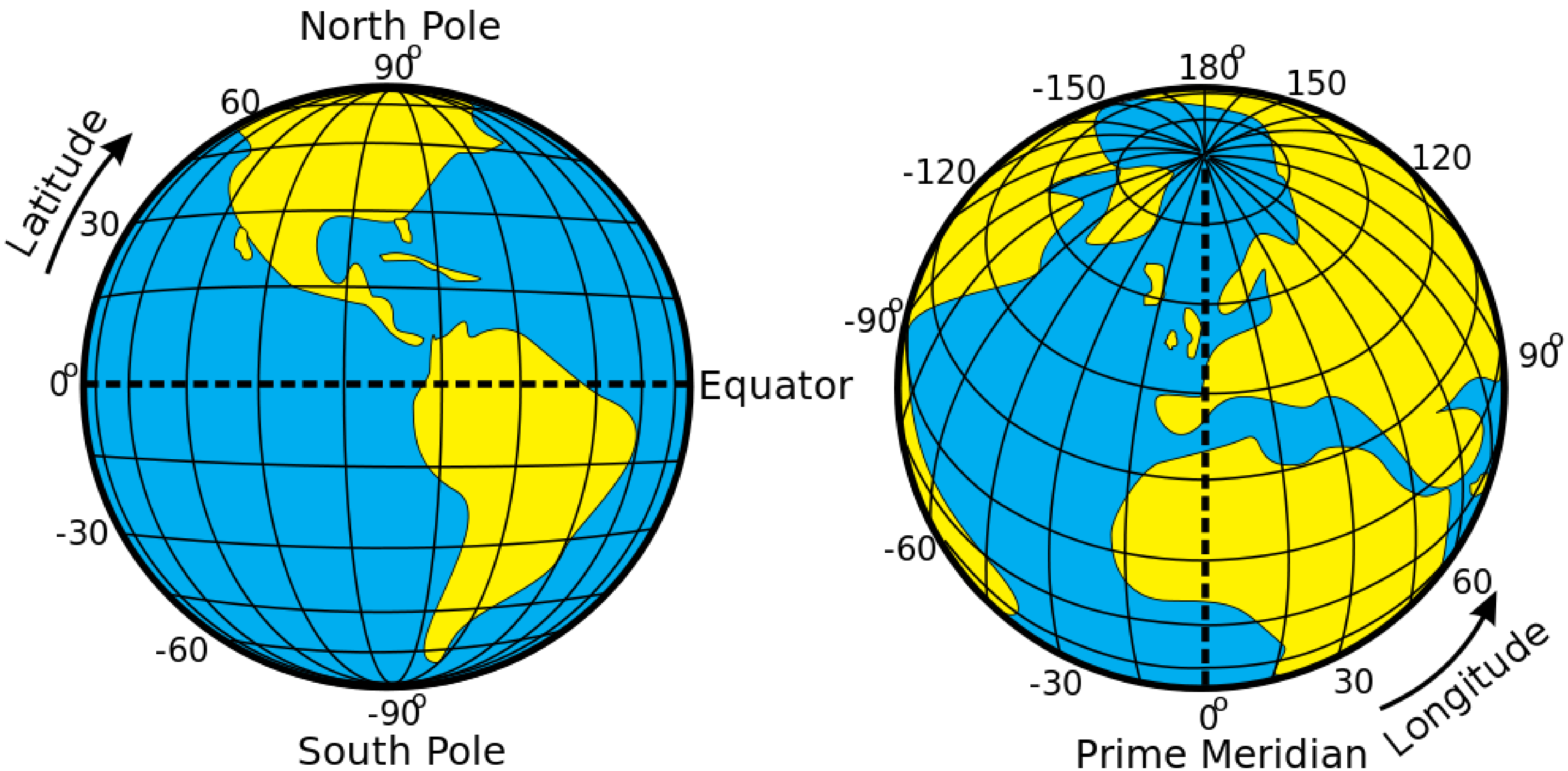
the apparent width of a dime 2.5 miles away

1/3,600 of a degree

2.8×10^{-3} deg
2.8 milli-deg

What are latitude and longitude?

Check to be sure you know!

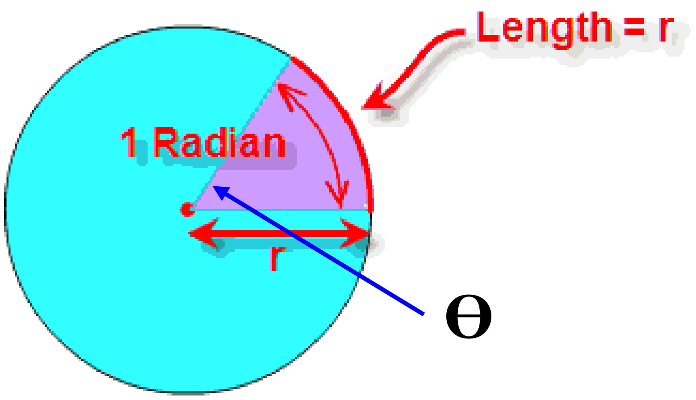


<u>Tucson</u>: 32.2226° N and 110.9747° W

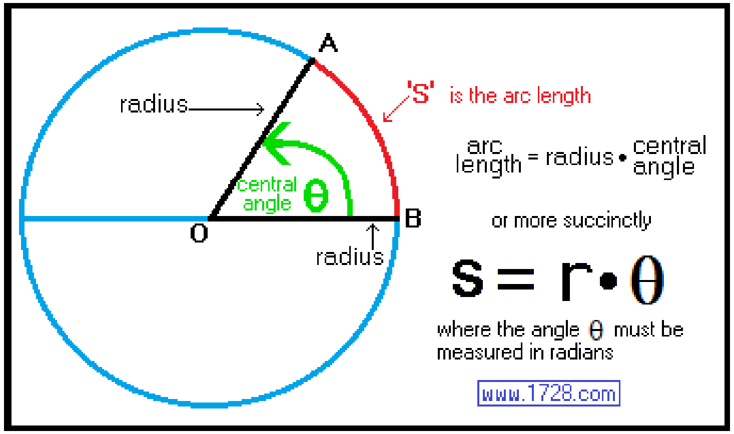
Develop a Plan: Convert these to deg: arcmin: arcsec

What is a ‘radian’ ?
another unit of angle

- One radian is the angle (ϴ) subtended by the arc of a circle with the same length as the circle’s radius.
  - a ratio of two lengths, i.e., dimensionless

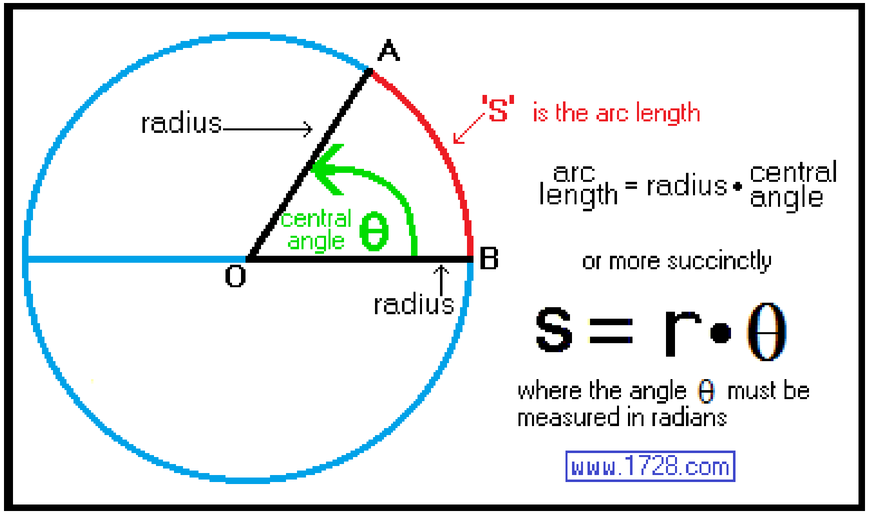


- In general, s = r ϴ, where ϴ is expressed in radians.

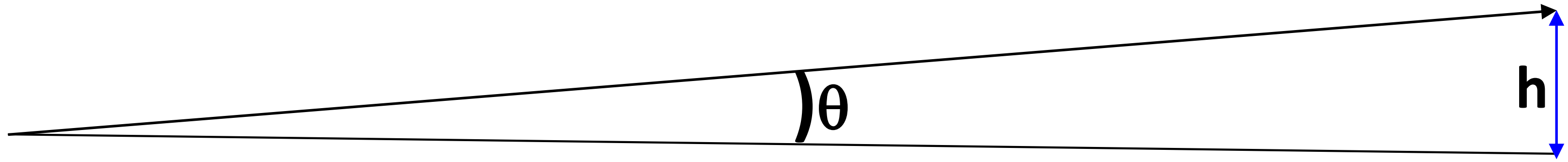


What is a ‘radian’ ?
another unit of angle

- For a “unit circle,” r = 1.
- Consider the circumference:
  - $\Theta = s/r = 2\pi r/r = 2\pi$ radians
  - conversion: 2π radians = 360 deg
  - $\theta = 360$ deg$/2\pi$
  - so, one rad = 57.3 degrees
- How many arcseconds per radian?



"Small Angle Equation"



Distance (D)

tan (θ) = h / D

when θ ≤ 10 deg, tan(θ) ~ θ in radians (≤ 1% error)

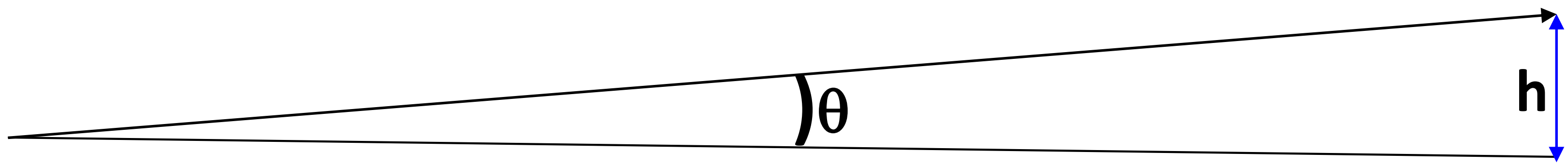
so, θ (rad) ≈ h/D

TRY IT! – Compare , tan(θ) to θ (rad)

But first: Can you switch your calculator between deg and rad?

Different rows: 10, 8, 6, 4, 2 degrees

"Small Angle Equation"



Distance (D)

$\tan(\theta) = h / D$

when $\theta \leq 10$ deg, $\tan(\theta) \sim \theta$ in radians ($\leq 1\%$ error)

so, θ (rad) $\approx$ h/D

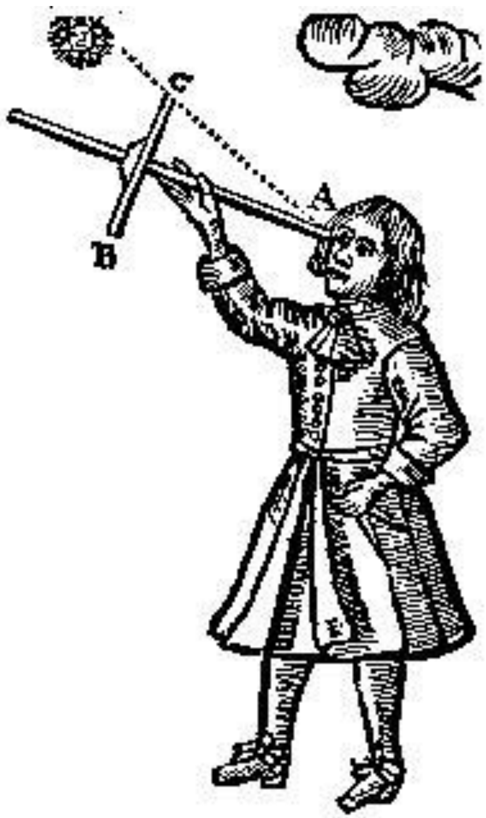
θ (deg) = θ (rad) * 57.3 deg/rad

θ (deg) ~ 57.3 * h/D

θ (arcsec) = 206265 * h/D

Problem: My Cross Stave

Why is a length of 57.3 cm convenient?



angle = 1 cm / 57.3 cm = 1/57.3 radians

angle = (1/ 57.3) rad x 57.3 deg/rad = 1 deg

<u>Or, visualize</u>:
One radian angle corresponds to 57.3 cm of arc and 57.3 cm of radius.

If you shrink the arc to 1 cm (i.e., 57.3 x), then the angle shrinks to 1 deg.

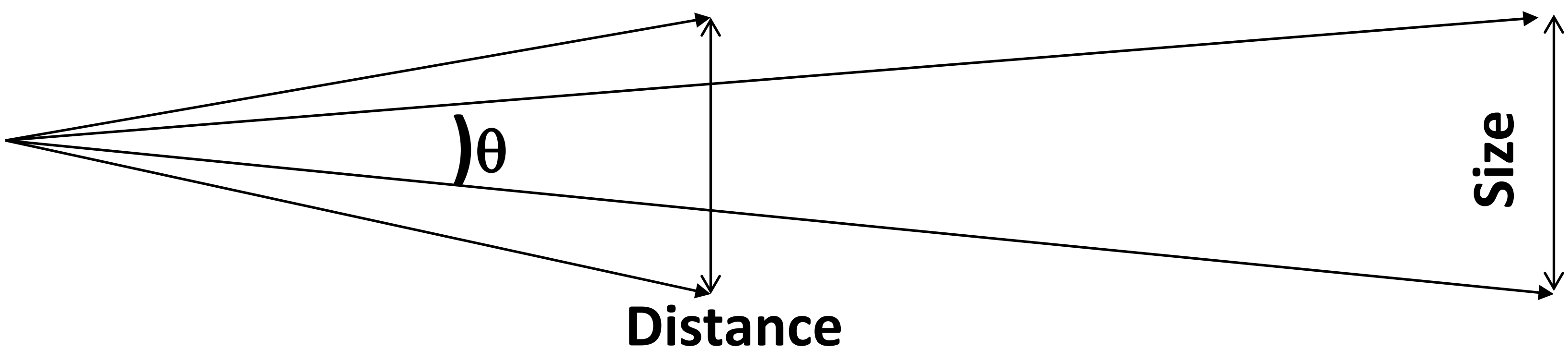
"Angular Diameter"
a ratio of size to distance

for small angles …

Size = Distance * Angle

$S = D * \theta$

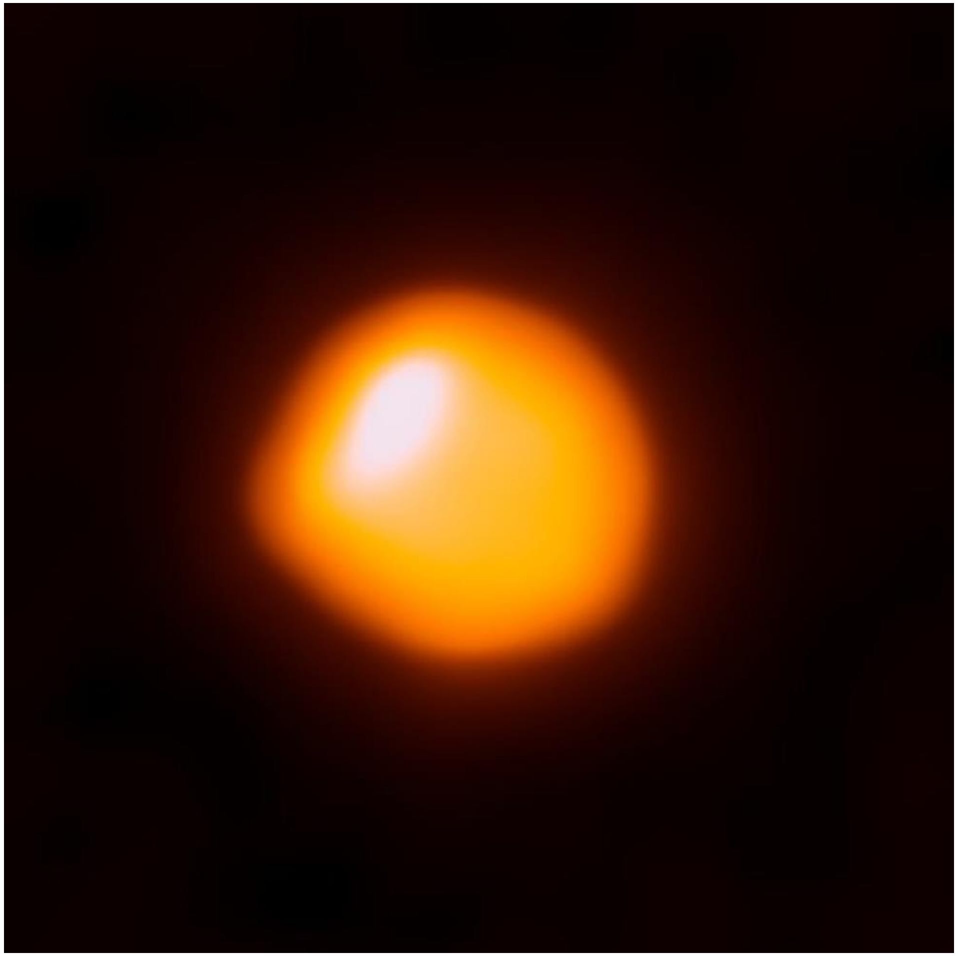
$\theta = S/D$ in radians



An object's "angular diameter" appears smaller the farther away.

Problem:

How would you estimate the angular diameter of the supergiant star Betelgeuse?



~10^3x larger than Sun
1.4 x 10^5 x farther than Sun

642 light-years away
1.2 x 10^9 km diameter

Problem:

First Proof of Earth's Motion

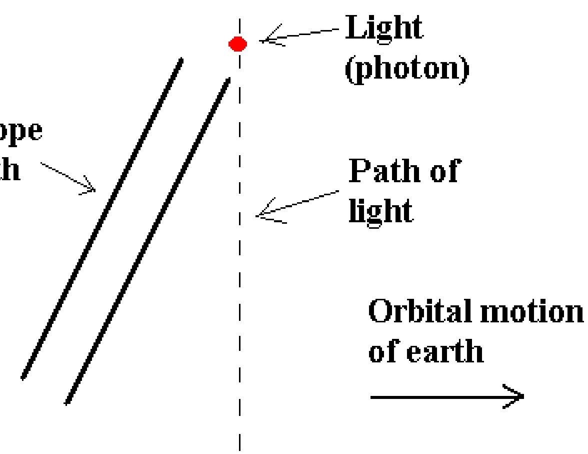
1728 – two centuries after Copernicus!

James Bradley (1693-1762)

<u>"Aberration of Starlight"</u>

- **Earth is traveling through a "wind" of starlight.**
- **The apparent direction of starlight shifts because of our motion.**
- **Earth moves 18.5 miles/sec.**
- **<u>How</u> determine the magnitude of this effect?**

Fig. 1 Aberration of starlight



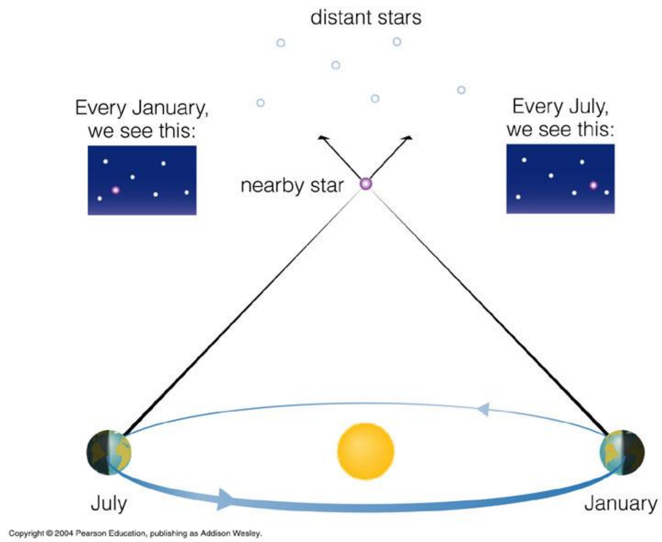
Measuring Distances to Stars
"parallax"

Easiest way of measuring distance: A surveying method

An object seems to change position if we change our viewpoint.

The Earth gives different viewpoints as it revolves in its orbit.

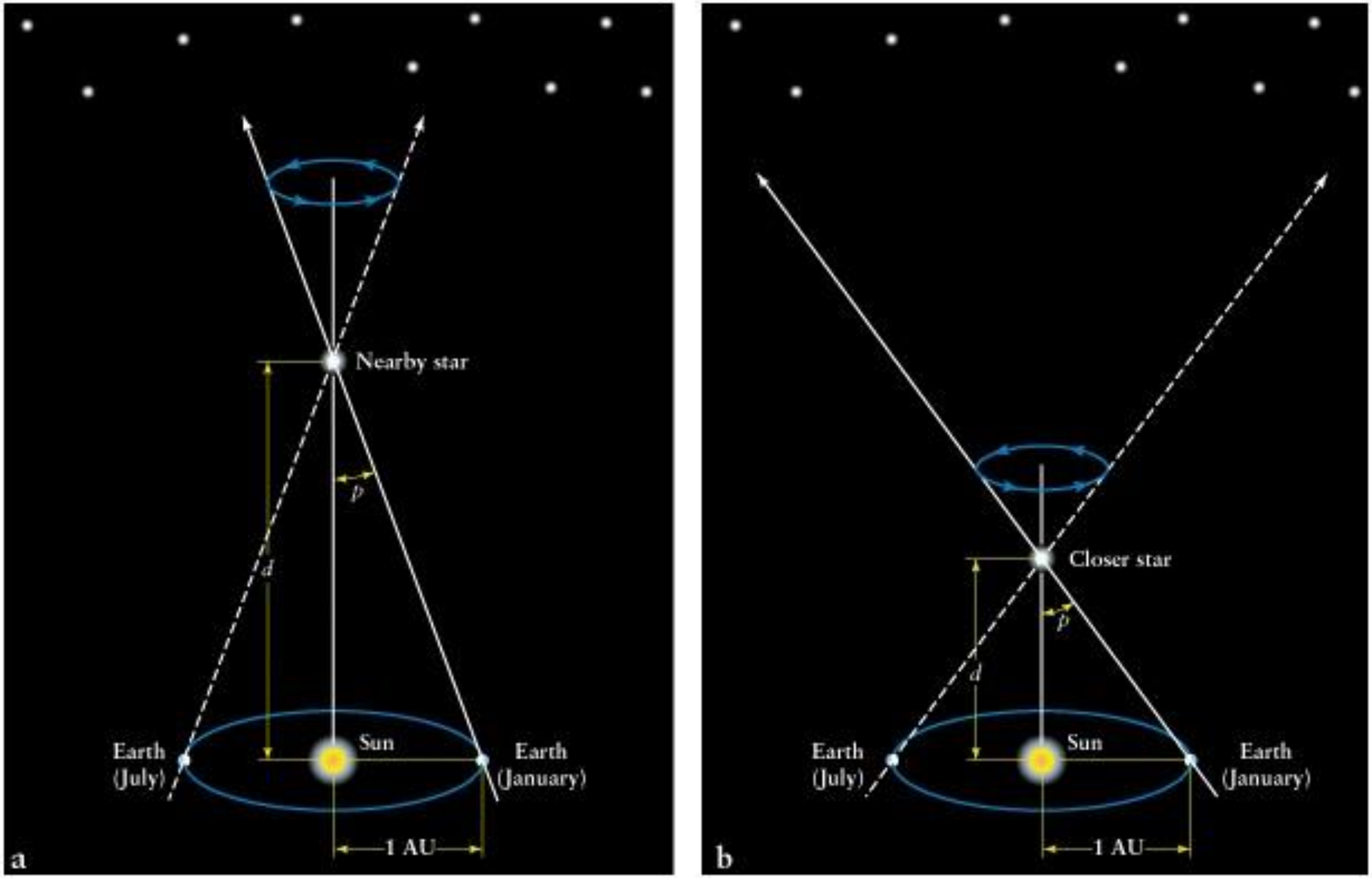
The angle a star appears to move is its "parallax."



Definitely NOT to scale!

WHY?

Why obtain the observations six months apart?



tan (p) = 1 AU / d

For small angles in radians, tan (p) ≈ p = 1/d

Parallax

What is a "parsec" (pc)?

- **p (radians) must be dimensionless**
  - **so d must be in AU**

- **p is actually measured in arcseconds, so**
  - **p (arcsec) = p (rad) x 206265 arcsec/rad**

- **p (arcsec) = 206265/d, where d (AU)**

- **p (arcsec) = 1/d where d is in "parsecs"**
  - **1 pc = 206265 AU = one parsec**

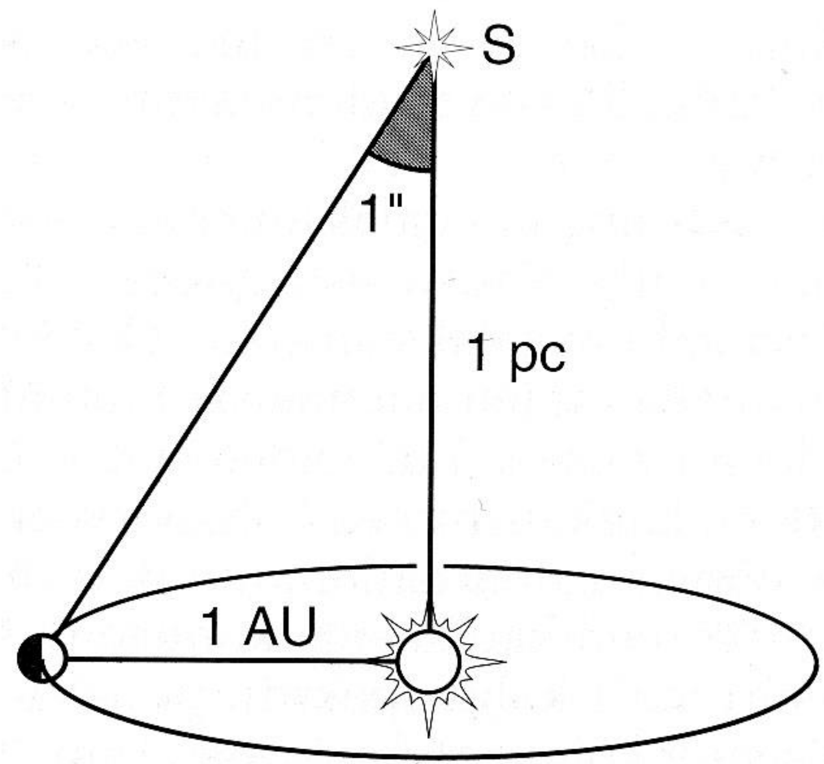
- **Parsec: <u>Par</u>allax of one arc<u>sec</u>ond**

A "Parsec"

corresponds to a <u>par</u>allax angle of one arc<u>sec</u>ond

A VERY SMALL ANGLE

(i.e., stars are far away)



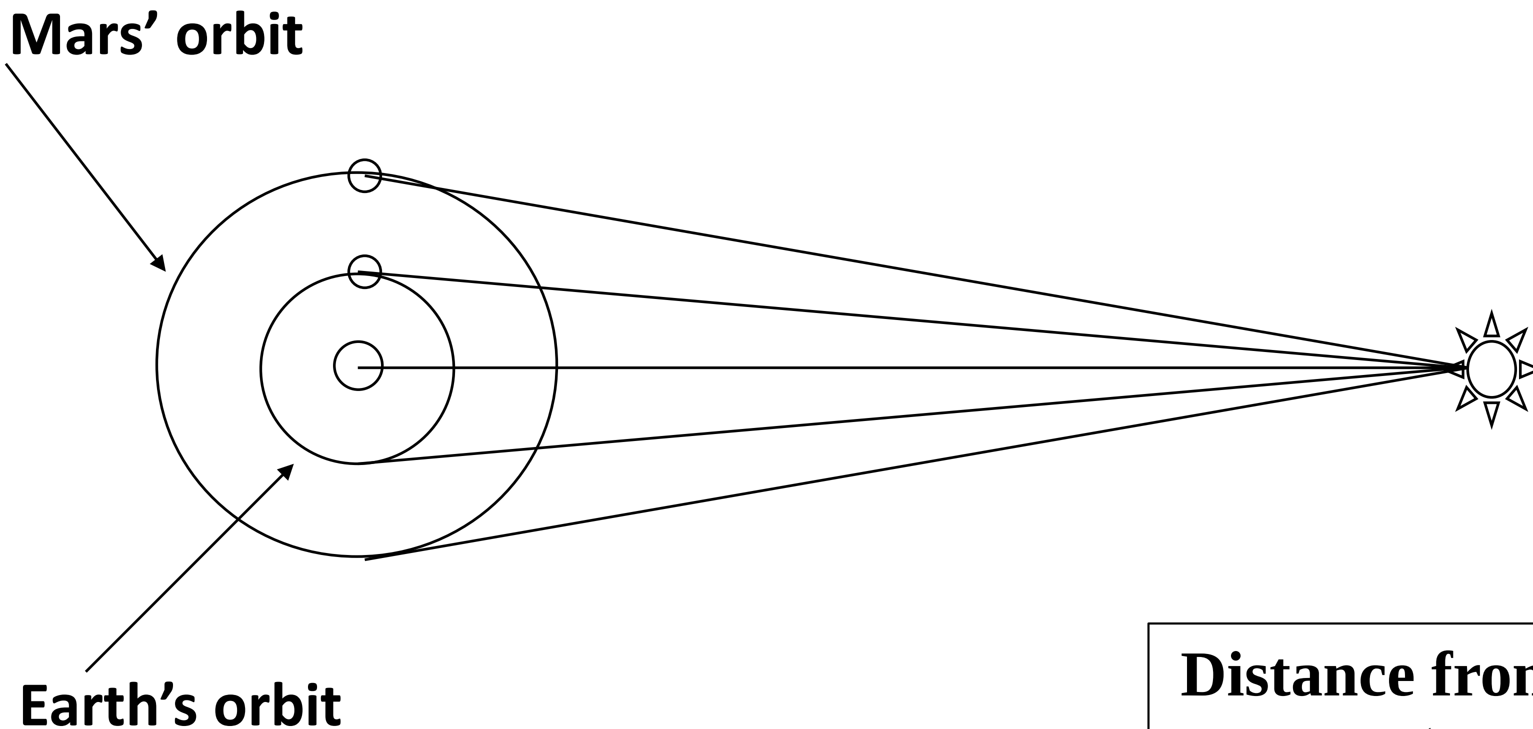
1 pc = 3.26 light-years

α Centauri: 1.3 pc (4.3 lyr)

$$\text{distance (parsecs)} = \frac{1}{\text{parallax (arcsec)}}$$

Problem:

<u>How</u> estimate how much bigger stellar parallax angles would be from Mars vs. Earth?



Distance from Sun:
Mars (1.5 AU)
Earth (1 AU)

Parallax angles to …

Across the diameter of Earth:
Moon (~1^{o})
Mars (~20“)
Sun (8.8”)

Across the diameter of Earth’s orbit:
stars (<1“)

Pointing a Modern Telescope

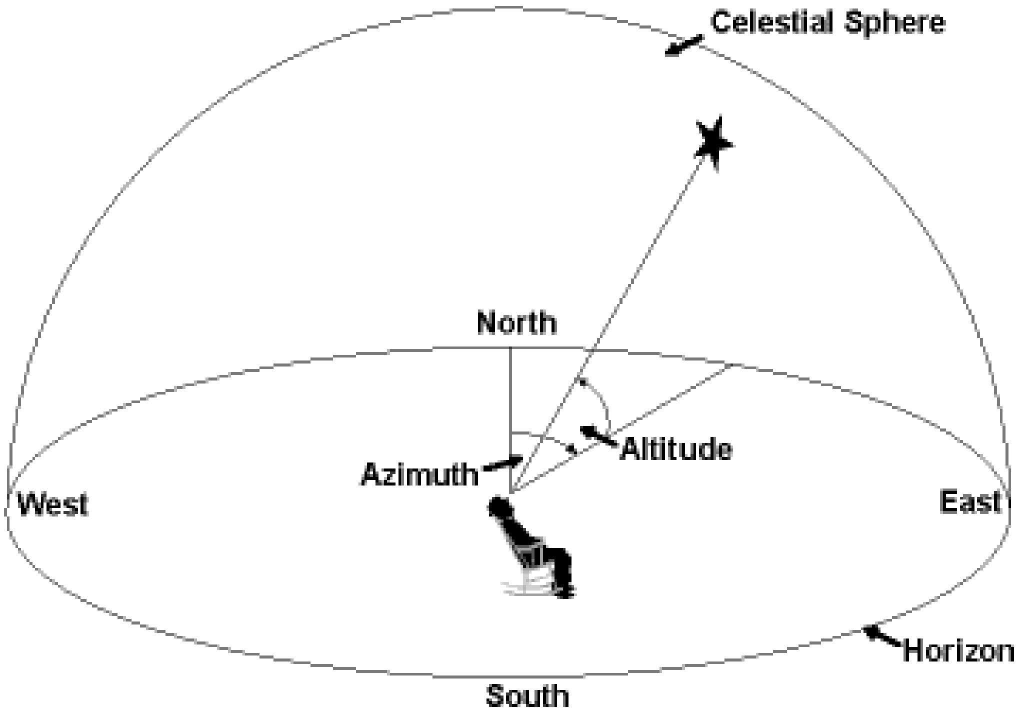
- **Must correct for these phenomena:**
  - **Aberration of starlight**
  - **Precession**
  - **Nutation**
  - **Refraction**
  - **Proper motion**
  - **Flexure**

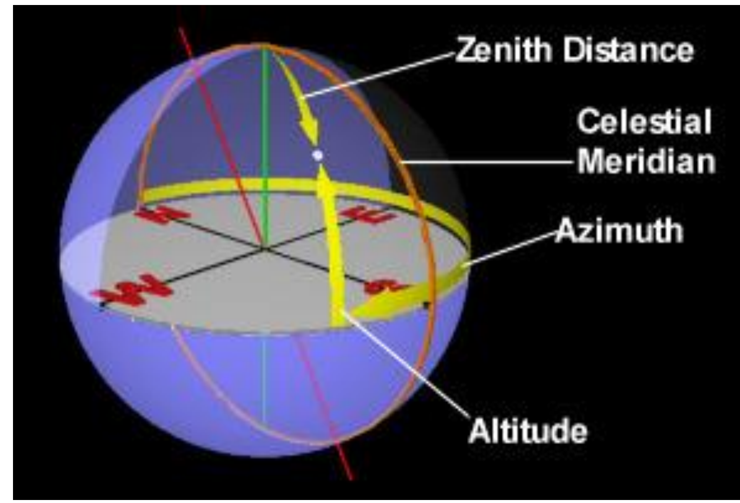
- **These corrections range from arcsec to arcmin.**

Azimuth, Elevation Coordinates

AZ-EL or AZ-ALT

change with time





<u>Terms</u>: Meridian, zenith, nadir, horizon

The 6.5 m MMT Observatory south of Tucson
All modern telescopes must point blindly to ~1 arcsec.

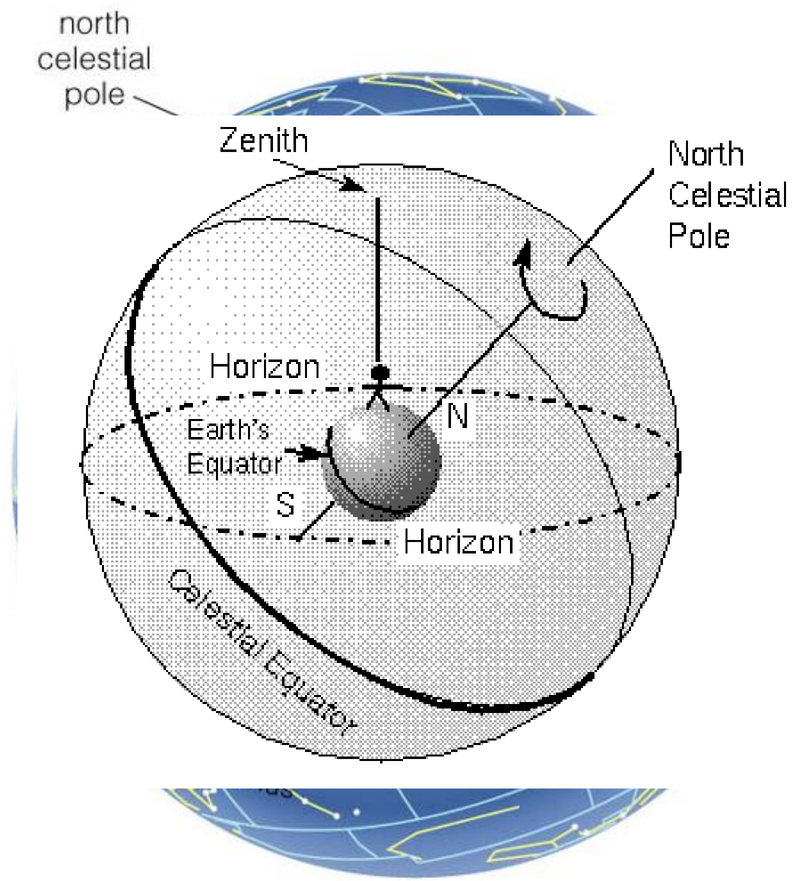
Equatorial Coordinates
the "Celestial Sphere" concept
mark on your plastic ball

Earth was imagined to be inside at center.

Stars & constellations are fixed on a rotating sphere surrounding the Earth.

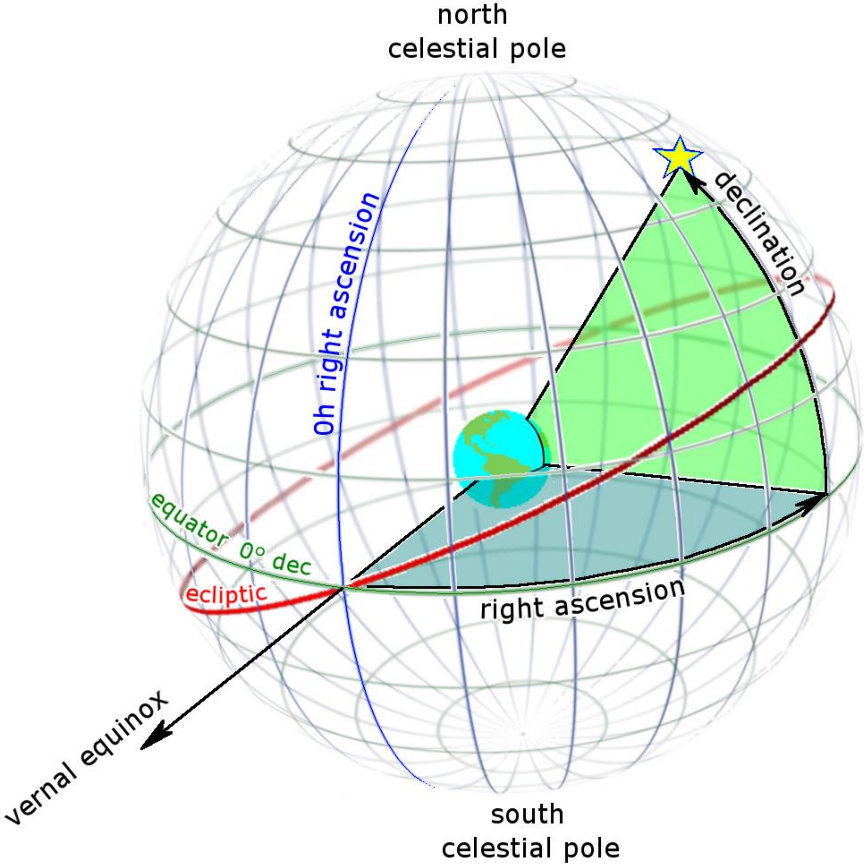
Earth's poles and equator are "projected out" onto the celestial sphere.

Sun moves along the yellow path ("ecliptic").



<u>Terms</u>: Celestial poles and equator, ecliptic

Equatorial Coordinates
right ascension and declination
do not change with time (sort of)



- **DEC: deg: arcmin: arcsec**
- **RA: hours: min: sec**
- **<u>How</u> convert between time and angle?**

Note how zero-point of RA is defined.

How convert between time and angle?

- **Earth rotates 360 degrees in 24 hours**
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Equatorial Coordinates
right ascension and declination
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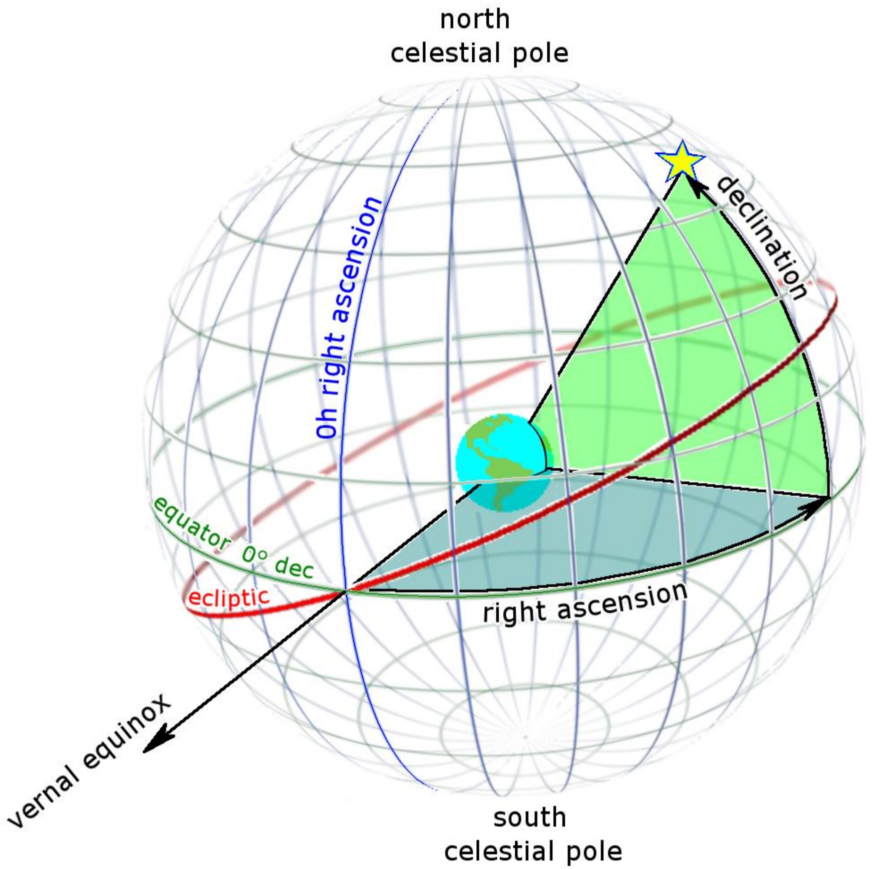
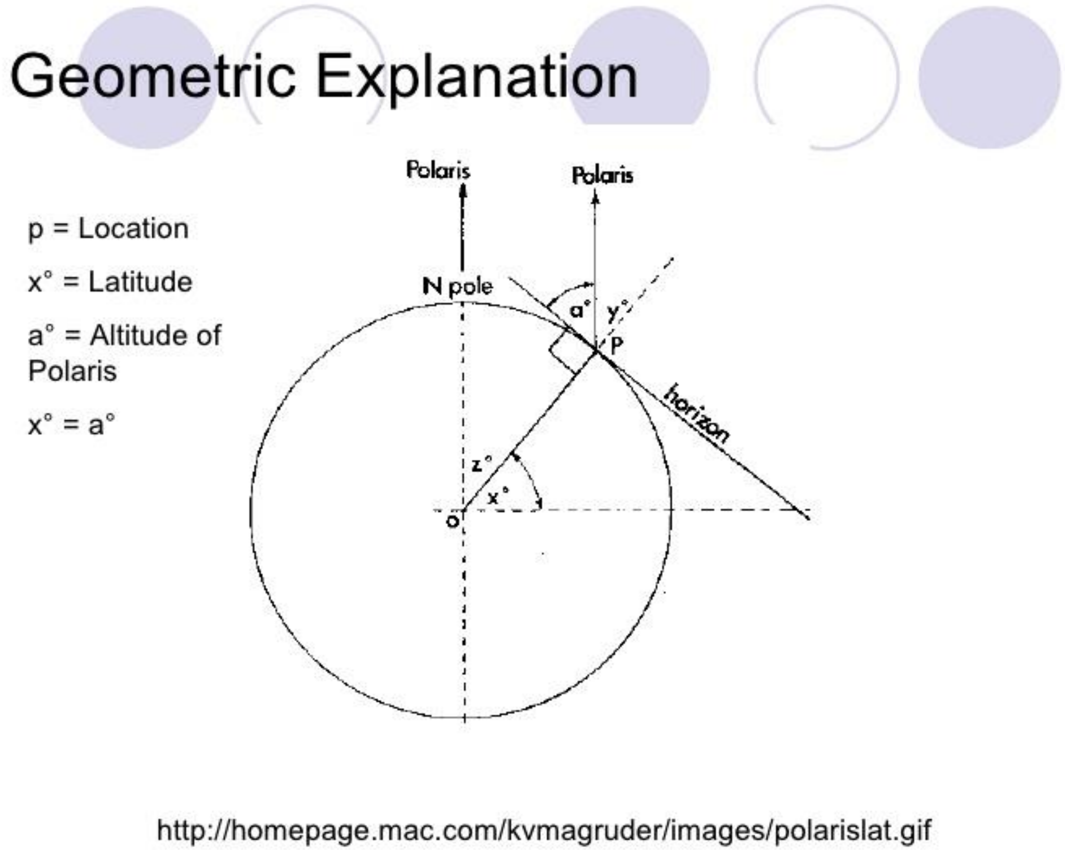


- **What is the declination of Polaris?**
- **From Tucson (lat ≈ 32 deg)**
  - **What is the altitude of Polaris?**
  - **What DEC is overhead?**
  - **What is the lowest DEC you can observe?**

Angle of Polaris Above the Horizon



The angle of Polaris above horizon (altitude) equals your latitude.

Equatorial Coordinates

<u>More Terms</u>: sidereal time, hour angle, transit