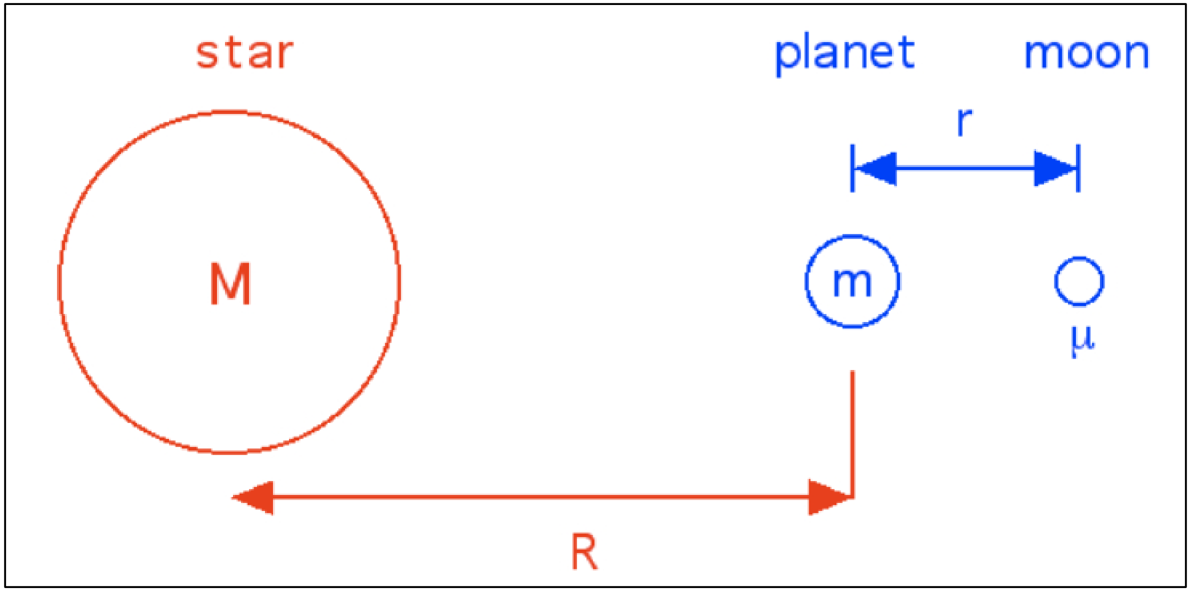
Hill Sphere - Reprise



Consider rotation and mutual gravity

The satellite or moon (mass μ) is orbiting the star (mass M) with the same angular velocity ω at the distance R+r as the planet (mass m) at the distance R (permanent full moon position).

The equilibrum condition for the planet is:

$$m\,\omega^2 R = G\,m\,M/R^2$$

$$\omega^2 = GM/R^3$$

The satellite is dragged by the combined gravitational forces exerted by the star and the planet:

$$\mu\,\omega^2 (R+r) = G\,\mu\,M/(R+r)^2 + G\,\mu\,m/r^2$$

Inserting ω^2:

$$G\,\mu\,M\,(R+r)/R^3 = G\,\mu\,M/(R+r)^2 + G\,\mu\,m/r^2$$

$$M\,(R+r)/R^3 = M/(R+r)^2 + G\,m/r^2$$

$$M\,(R+r)^3\,r^2 = M\,R^3\,r^2 + m\,R^3\,(R+r)^2$$

$$m\,R^3\,(R+r)^2 = M\,r^2\,(R^3+3R^2r+3Rr^2+r^3) - M\,R^3\,r^2$$

$$m\,R^3\,(R+r)^2 = M\,r^3\,(3R^2+3Rr+r^2)$$

For $r<<R$: $(R+r)^2 \approx R^2$, and $3Rr+r^2 \approx 0$. The equation simplifies:

$$m\,R^5 = 3\,M\,r^3\,R^2$$
$$m\,R^3 = 3\,M\,r^3$$

$$\mathbf{r = R\,[m/(3M)]^{1/3}}$$

Hill Radius ("Sphere")

"instability limit"

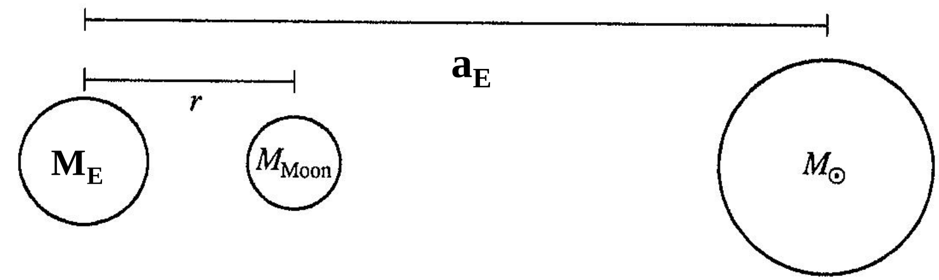
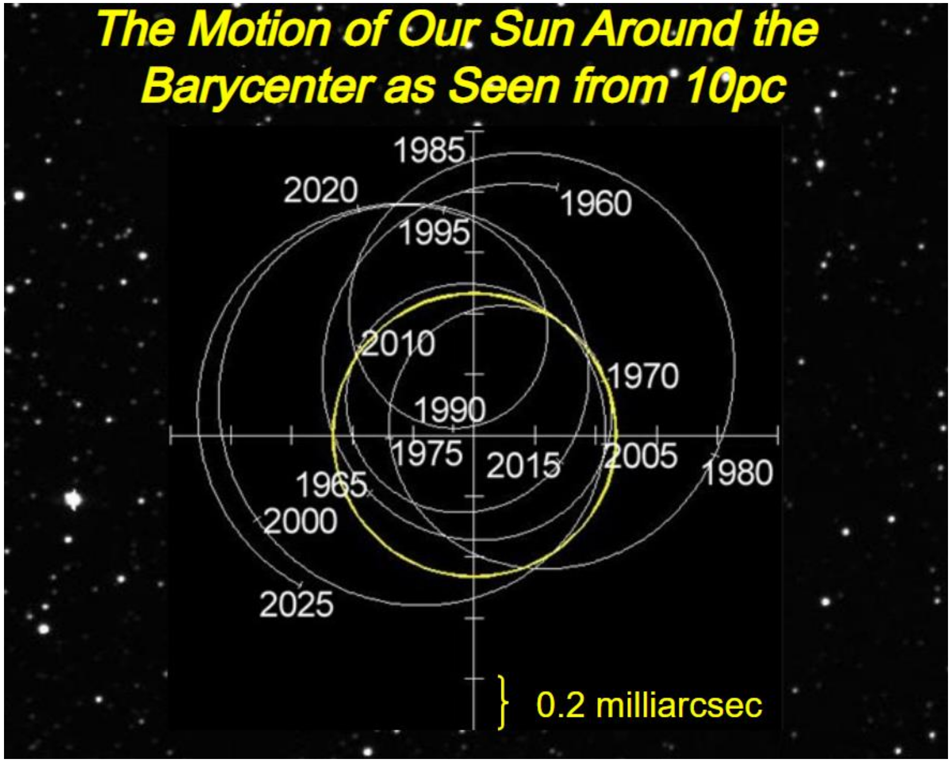


FIGURE 4.9 The Moon and the Earth, with masses M_{Moon} and $M_{\oplus}$, respectively, form a gravitationally bound system with separation r. The Sun, with mass $M_{\odot}$ at a distance $a_{\oplus}$ from the Earth, perturbs the Earth–Moon system. (Not to scale.)

What approach to this equation?

$$d_H \approx (M_E / 2M_{\odot})^{1/3}\; a_E$$

Indirect Method #1
astrometry



Jupiter's ~12 yr orbit causes Sun to move ~0.5 milliarcsec.

Predict the Earth's effect.

$$M_{star}\, a_{star} = M_p \;\; a_p$$

For a given star,
amplitude α M_p a_p

For our solar system viewed from 10 pc away:

Planet	Orbit Size	Angular Size	Period
Jupiter	5.2 AU	0.5 milliarcsec	11.9 yr
Uranus	19.2 AU	84 microarcsec	84 yr
Earth	1 AU	0.3 microarcsec	1 yr

1.0 milliarcsec is 1/3,600,000 degrees!

The current ground-based limits are a few milliarcsec.

To do better, we must use a spacecraft.

So far, there are no good, reliable detections of extrasolar planets via astrometry.

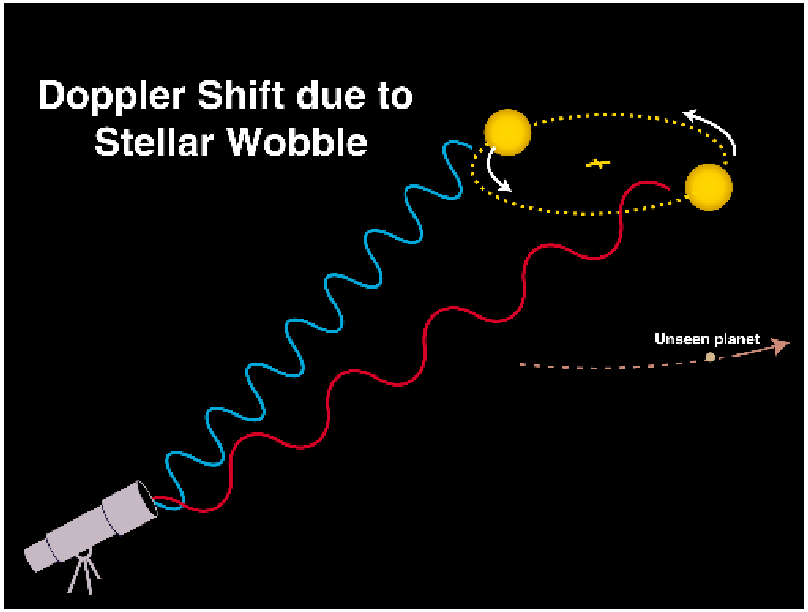
Indirect Method #2
Doppler shift and spectroscopy

Center-of-Mass:

planet and star

both objects move around it

The Sun moves 10 meters/sec and takes 11 years to make one full circle.



$$\Delta\lambda / \lambda = v_r / c$$

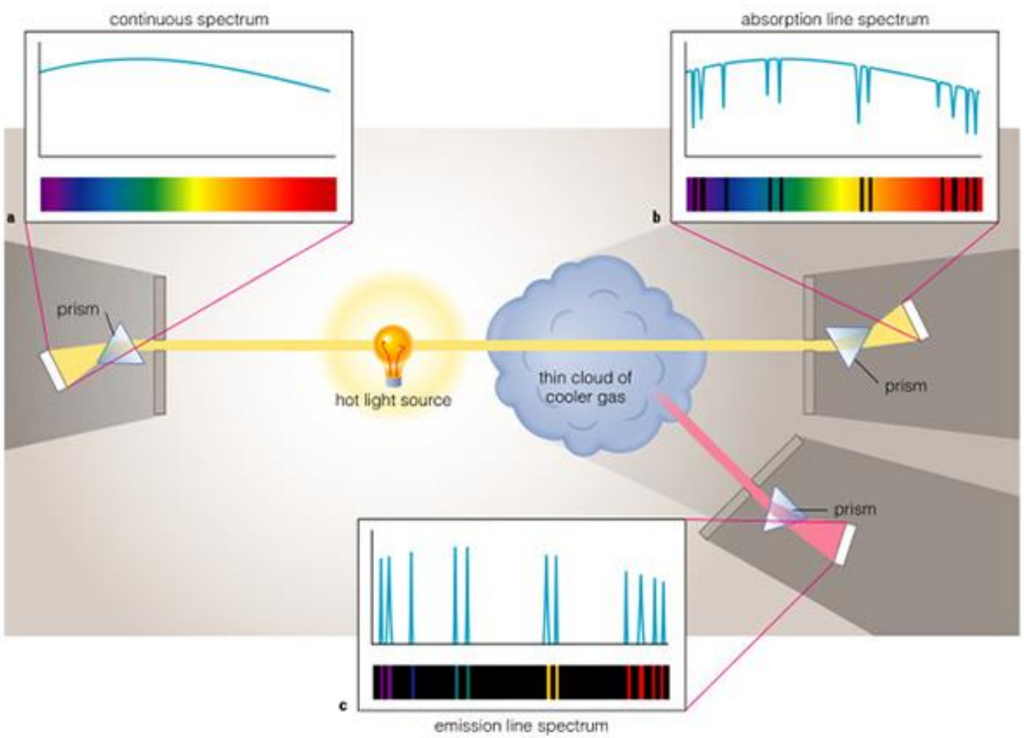
Problem

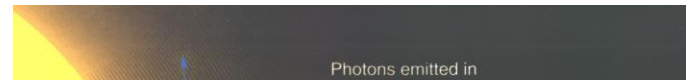
Some of today's exoplanets were discovered by precise monitoring of the Doppler Effect in visible light from the parent stars. The Doppler Effect causes a change in wavelength ($\Delta\lambda$) proportional to v/c, where v is the star's velocity back-and-forth caused by the planet's gravity.

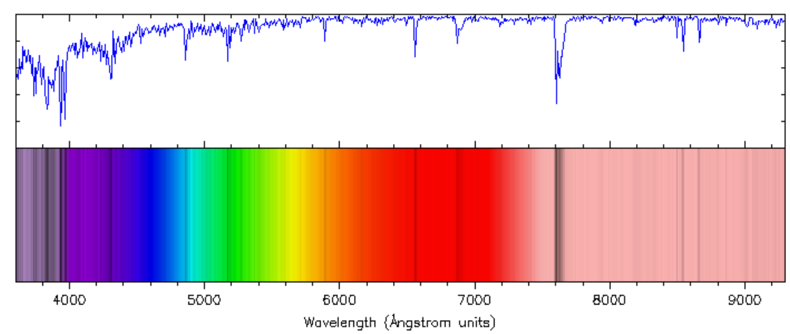
$$\Delta\lambda/\lambda = v/c$$

Today's technology can measure velocities of <1 $msec^{-1}$. What is the resulting wavelength shift ($\Delta\lambda$)? Are you surprised by this answer; why or why not?

Spectroscopy

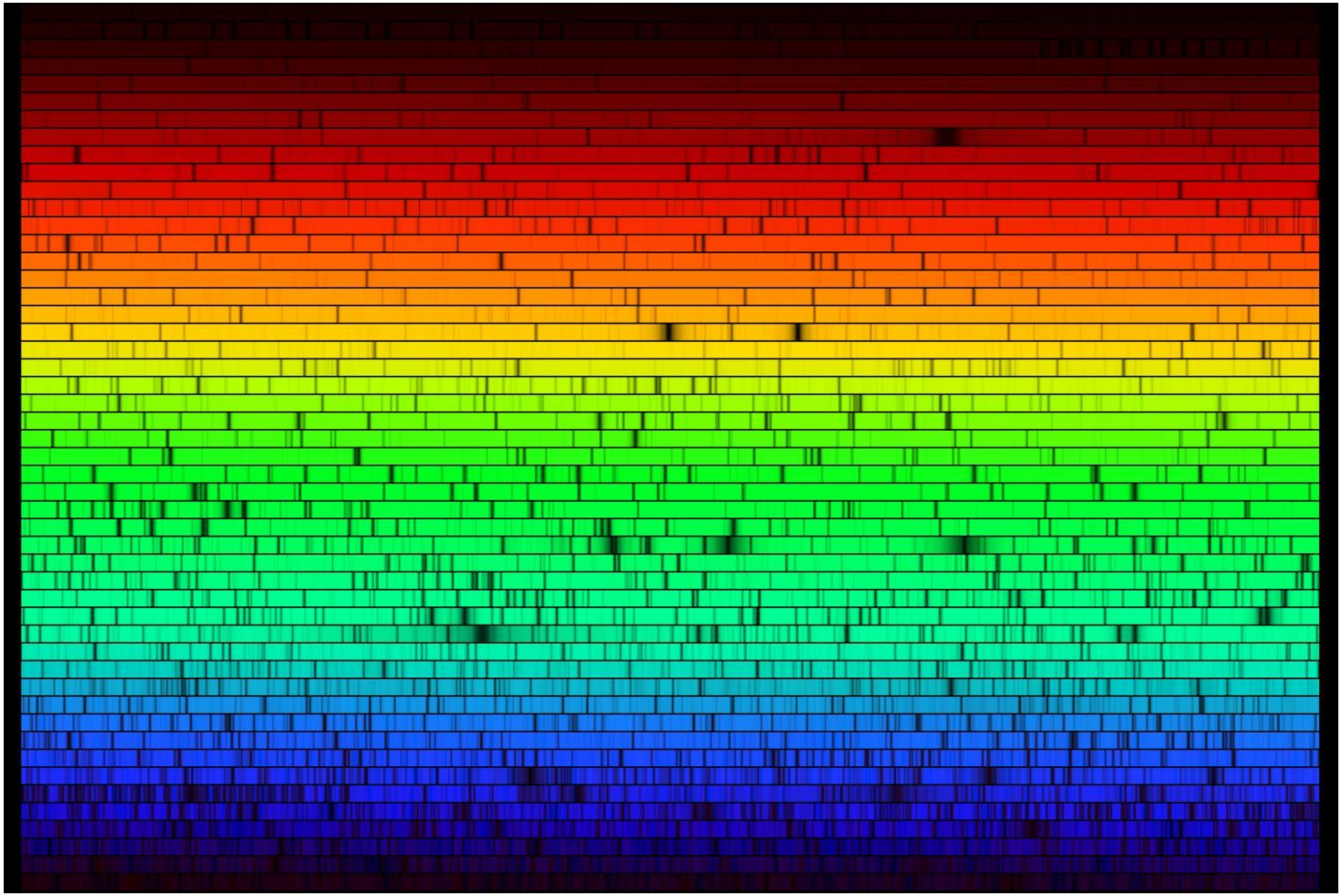






Data from "Photometric Atlas of the Solar Spectrum from 3000 to 10,000 A" by L. Delbouille, L. Neven, and C. Roland
Institut d'Astrophysique de l'Universite de Liege, Observatoire Royal de Belgique, Liege, Belgique, 1973

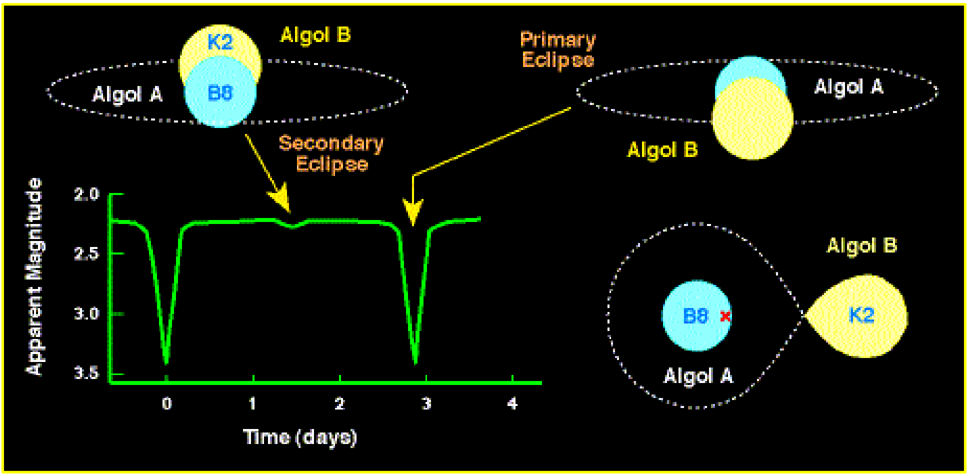
Solar Spectrum

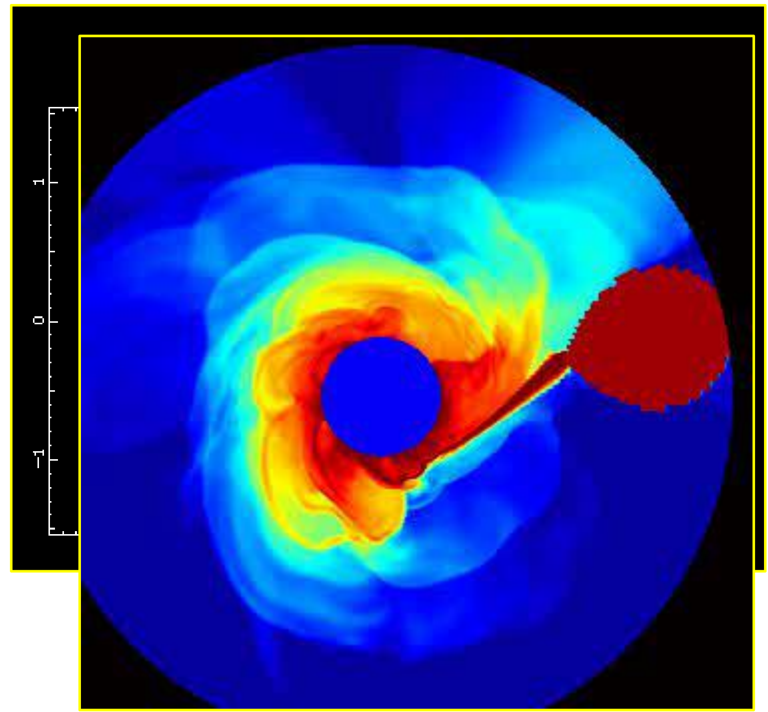


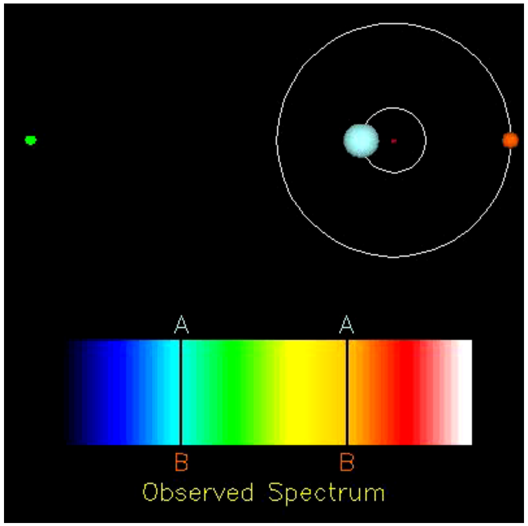
Algol (β Perseus)

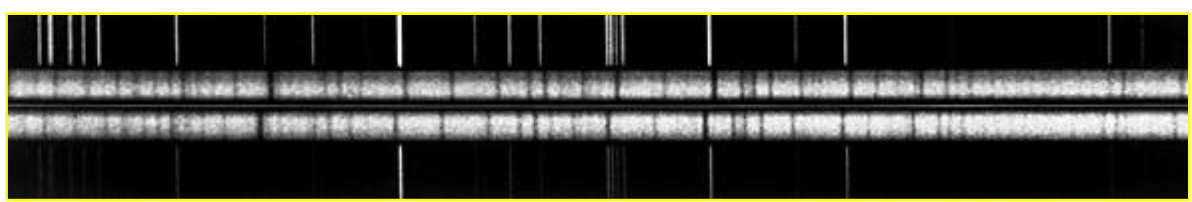
eclipsing binary (2.86739 days)

distance: 92 light-years = 28 parsecs



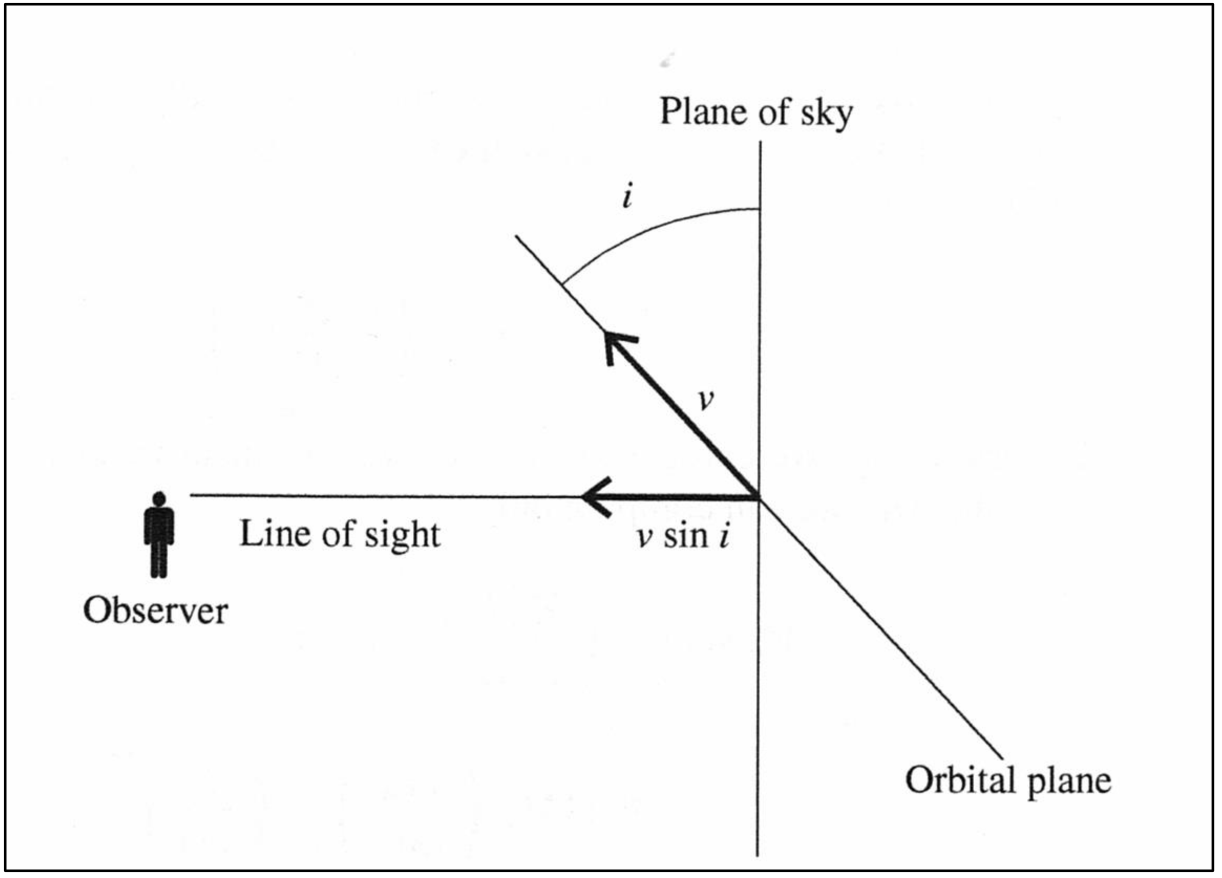






movie

Projected Radial Velocity



Kepler's 3rd Law
equation (12.21)

$$M_p \sin i \approx 11\ M_e\ (M_s)^{2/3}\ P^{1/3}\ (v_s \sin i)$$

P (years), M_s (M_{sun}), v (m/sec)

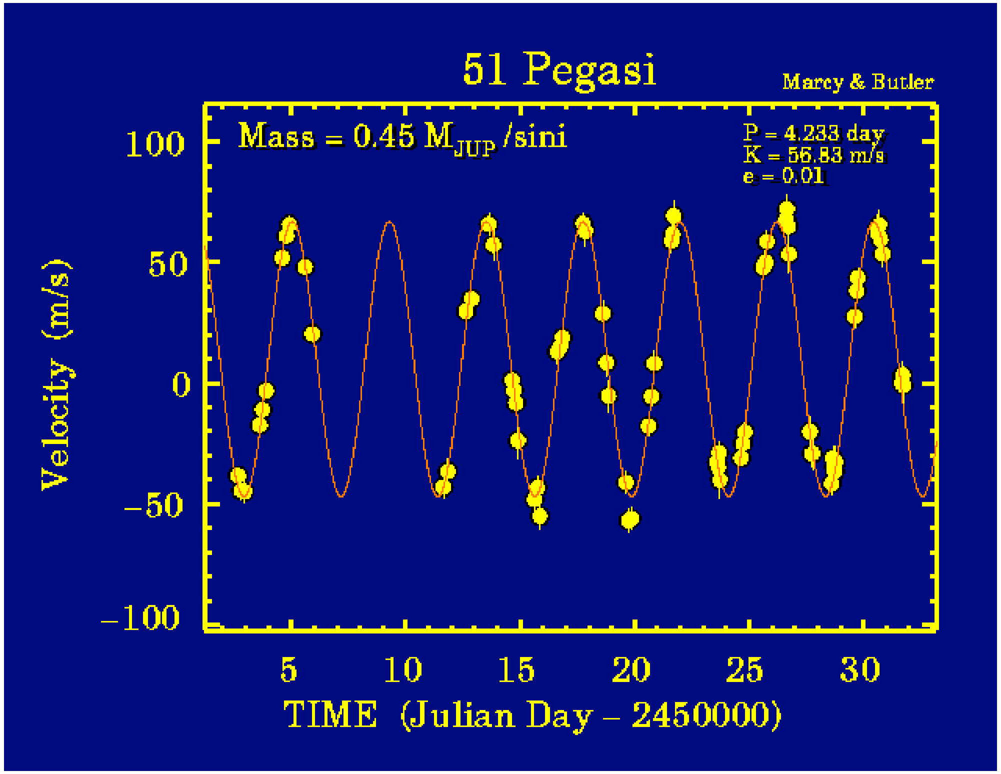
$$v_s\ \alpha\ M_p M_s^{-2/3}\ P^{-1.3}$$

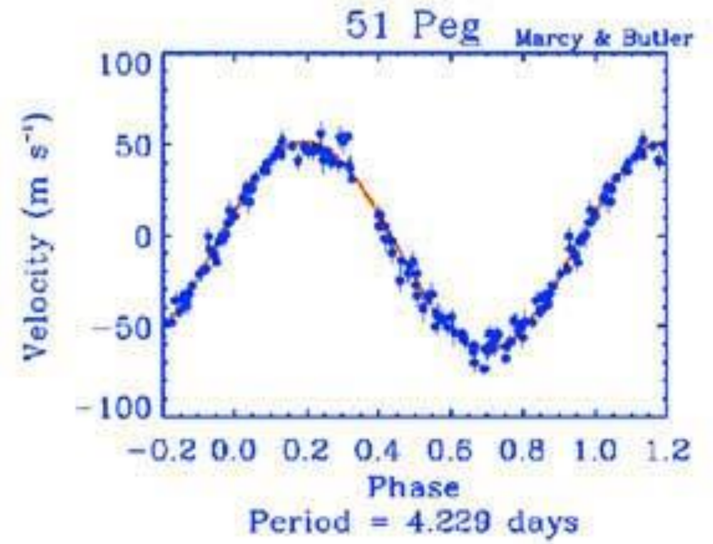
What characteristics of exoplanets would promote detection by the Doppler Effect?

The First Extra-Solar Planet

1995: 51 Pegasus b

Do you notice anything unusual about this graph?





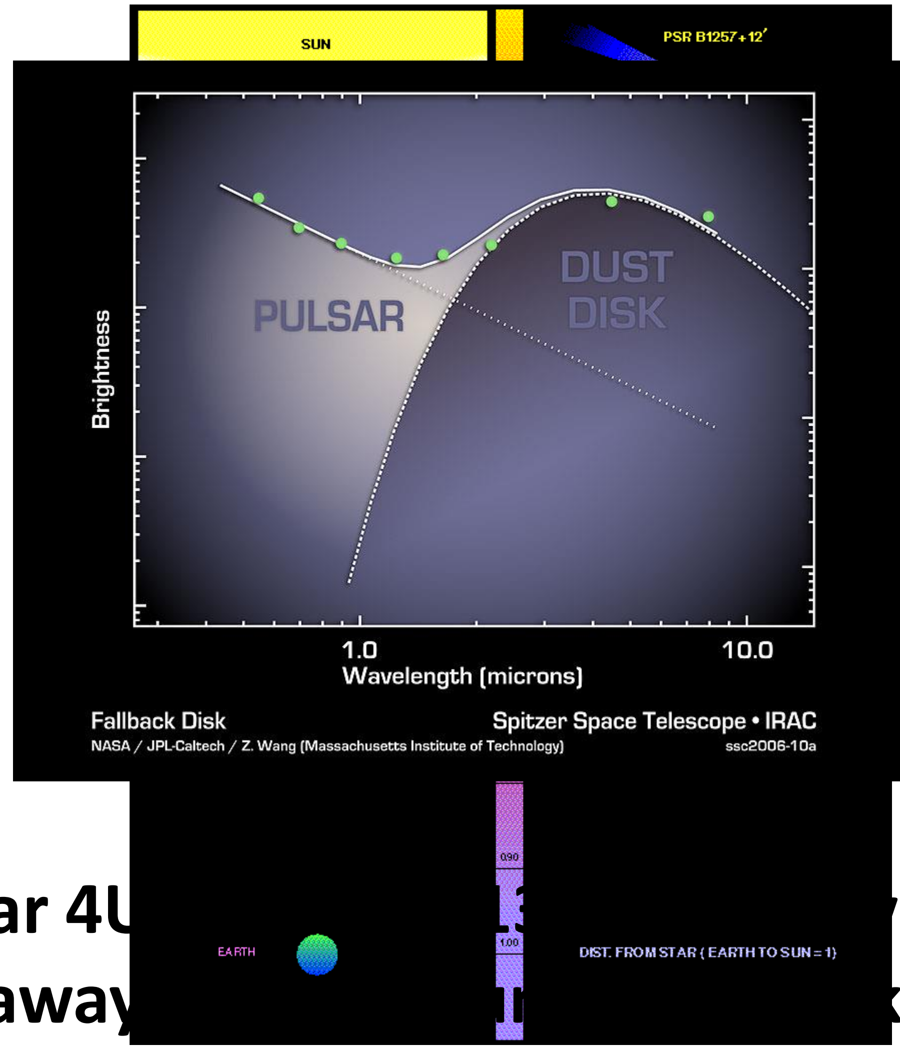
Pulsar Planets (PSR B1257+12)

the first extra-solar planets were discovered around a dead star

Detected via the timing of pulses from the pulsar

Second Generation Planets?

0.02, 3.9, 4.3 M_{Earth}



Pulsar 4U ... ears away ... k

Easier to measure change in time than distance

- $\Delta\lambda/\lambda$ = v/c
- If v =1 m/sec, then $\Delta\lambda/\lambda$ = 1/c

- From homework #5:
  - $(c/\nu)\, d\nu = d\lambda/\lambda$
  - $d\nu/\nu = (1/c)\,(v/c) = v / c^2$

- If v = 1 m/sec, then $d\nu/\nu = 1/c^2 = 1 \times 10^{-17}$

- The pulsar's frequency = 4.5×10^3 Hz
  - speed of 1 m/sec corresponds to $d\nu = 4.5 \times 10^{-14}$

- In 1992, atomic clocks achieved accuracy of $d\nu = 3 \times 10^{-16}$

Problems
exoplanet: 47 UMa b

- **47 UMa A**
  - **1.48 L_{sun} and 1.08 M_{sun}**

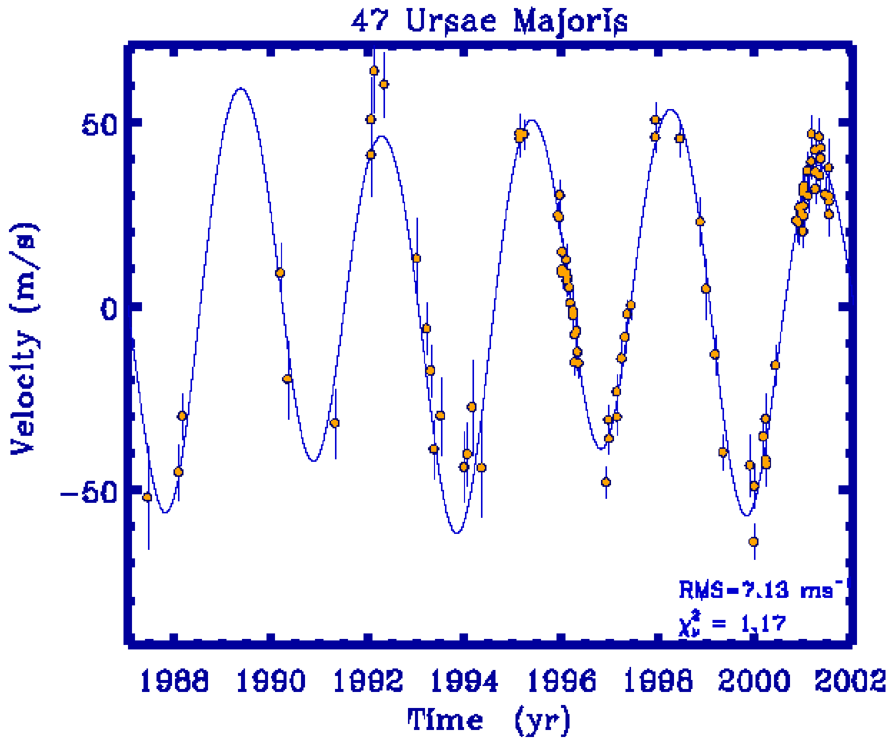
- **Assuming the period of the orbit from the velocity curve is 2.95 years, calculate the planet's semi-major axis (in AU).**
  - **In habitable zone?**

- **From equation (12.17),**

  $a_B \approx (1\ AU)\ M_A^{1/3}\ P^{2/3}$, where M_A is in solar masses and P is in years

- **Estimate the minimum mass of the planet (quote as a ratio to the mass of Earth).**

- **$M \sin(i) = 11\ M_{Earth} \cdot M_A^{2/3}\ P^{1/3}\ v_A \sin(i)$**



Indirect Method #3

eclipses ("transits")

probabilities: 0.5% (Earth-like); 10% (Jupiter-like)

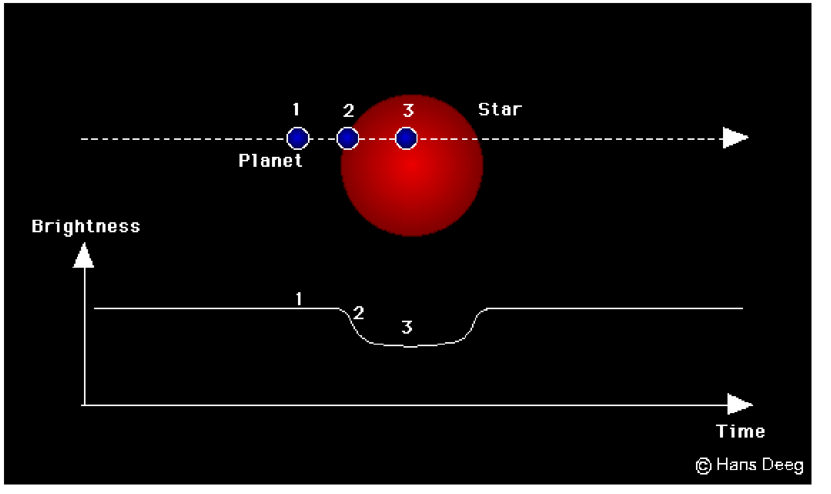
A planet can cross in front of a star diminish some of the star's light.

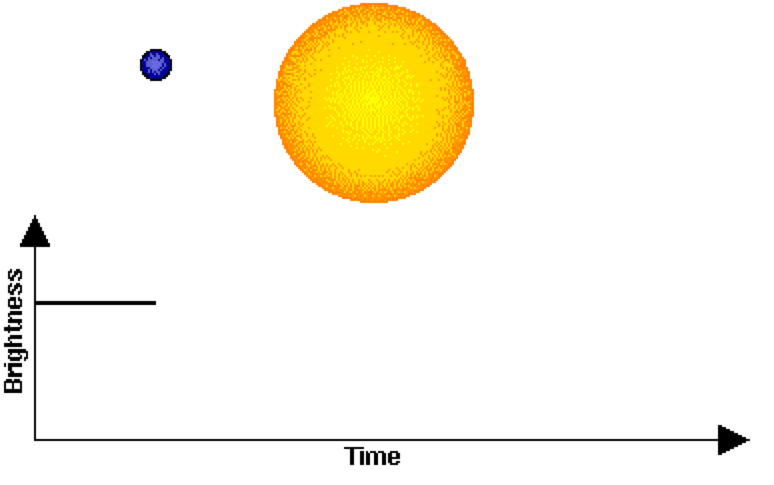
How does the drop in flux scale with diameter of the planet?

What percentage signal drop do you expect?

Sun-Jupiter= ?

Sun-Earth = ?



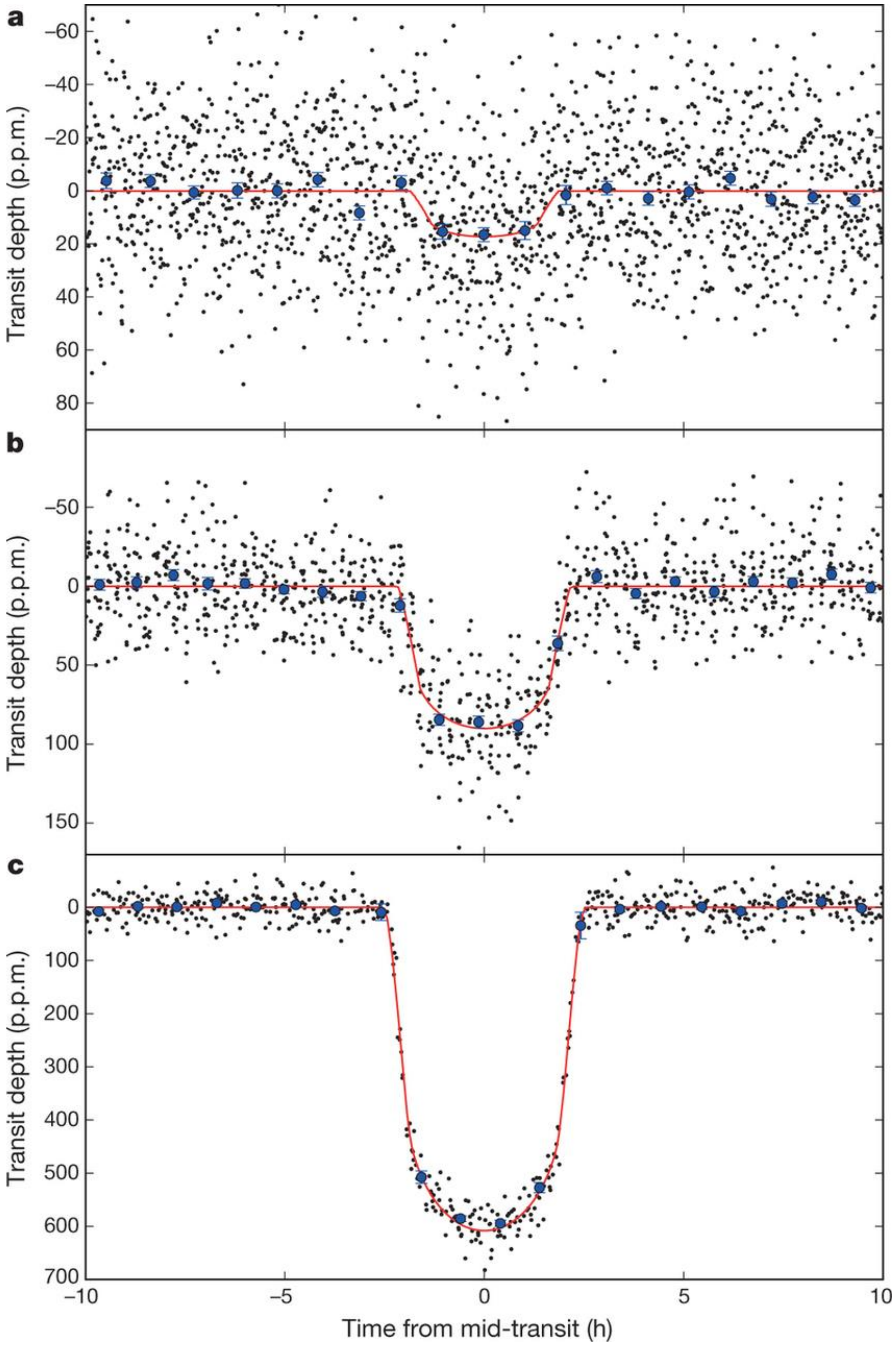


Solution

- **Fractional change in area**
  - **(AREA – area) / AREA**
  - **$(\pi R^2 - \pi r^2) / \pi R^2$**
  - **$1 - (r/R)^2$**

- **$R_{Jup} \sim 0.1\ R_{Sun}$**
  - **So, Jupiter would reduce the Sun's light by 1%.**

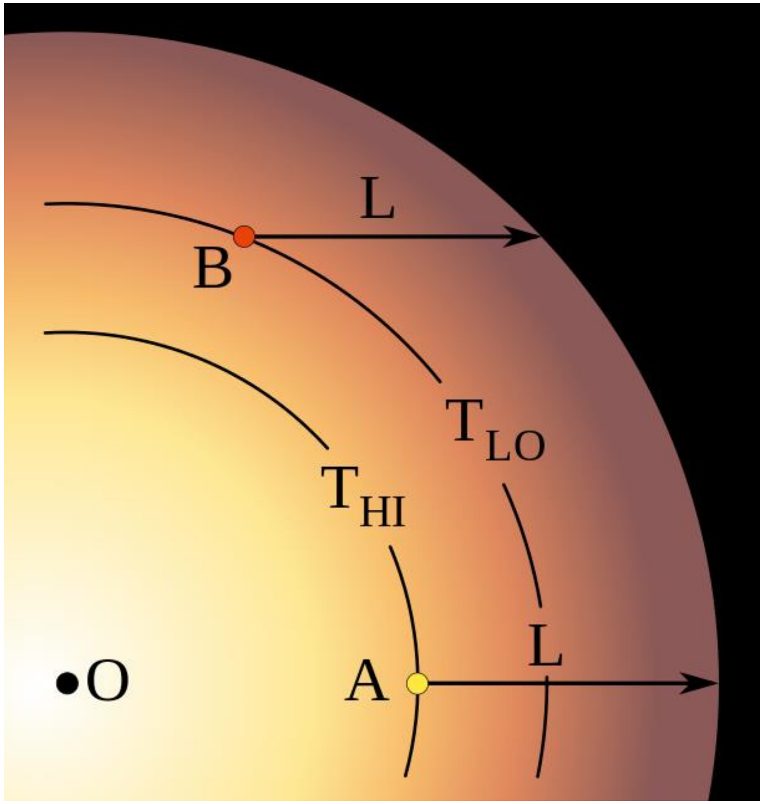
- **$R_{Earth} \sim 0.01\ R_{Sun}$**
  - **So, Earth would diminish Sun's light by ~0.01 %.**
  - **1 part in 10^4 = 100 ppm**
  - **duration of expected eclipse ~2-16 hours.**

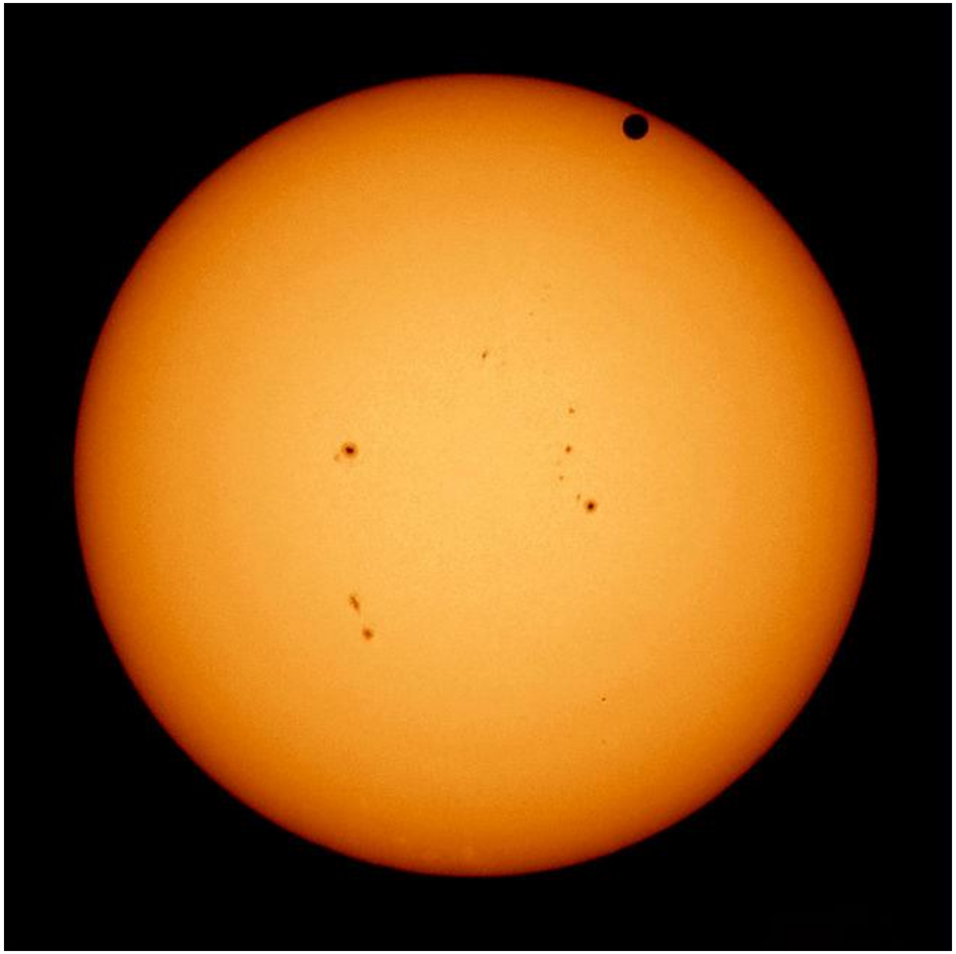


Kepler 37 b,c,d planets

R_{Earth} =
0.35
0.74
2

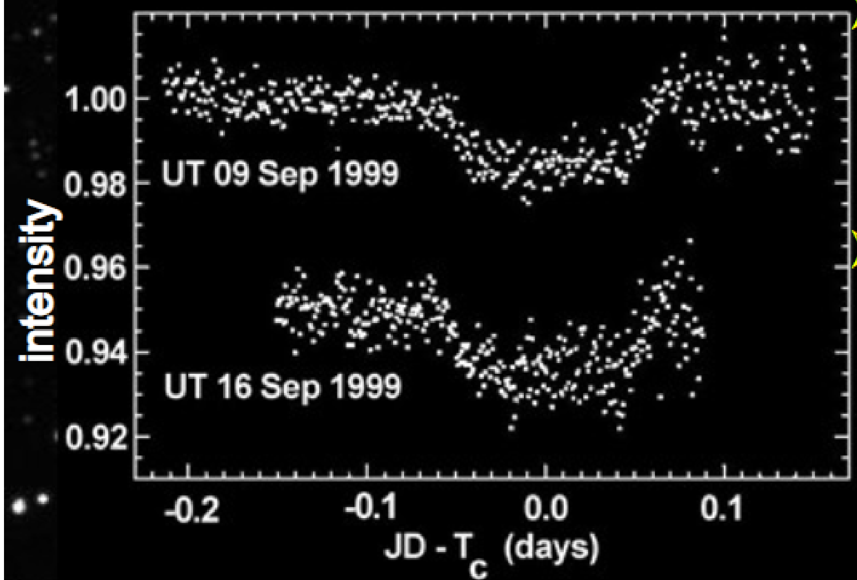
Limb Darkening of Stars





The Planet Around Star HD 209458

Charbonneau *et al.* 2000



- The planet around HD209458 was discovered by the radial velocity signal.
- After discovery, a transit was observed ⇒ this gives a good measure of the *inclination*. From M sin *i*, we can then derive M.
  - $M_P = 0.69 \pm 0.05\ M_{JUP}$
  - $R_P = 1.40 \pm 0.17\ R_{JUP}$
  - density = $\rho = 0.31 \pm 0.07\ g/cm^3$

Definitely a gas giant planet like Jupiter!

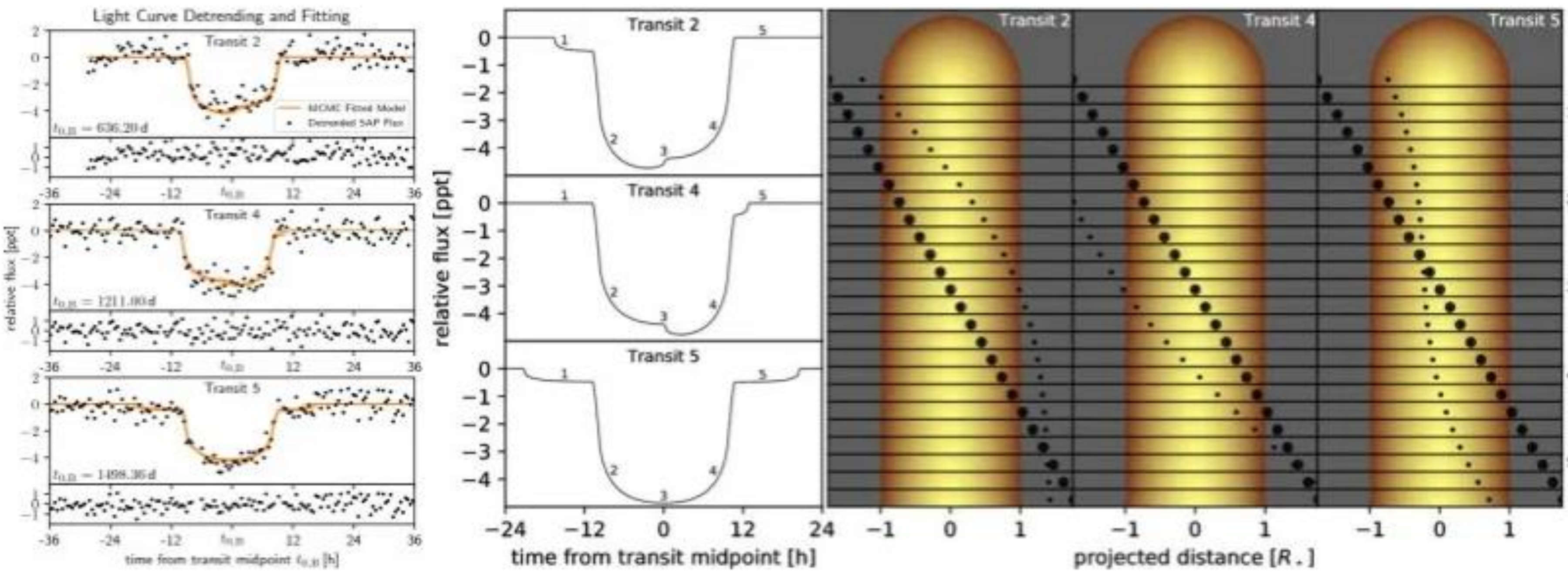
Kepler-1625b
transit detection of exomoon ?
observers followed up with HST instead of Kepler

- **Location:**
  - Cygnus
  - RA ~19:41 and DEC ~40^{o}

- **Star**
  - m_v = 13.9
  - distance 7181 pc
  - mass ~0.96 M_{Sun}

- **Exoplanet**
  - mass 3180 M_{Earth}
  - radius = 6-11 R_{Earth}
  - a = 0.8 AU
  - HST transit time occurred earlier by 77.8 min

Kepler-1625b
first detected exomoon?



Kepler Mission

- **The objective was a combined differential photometric precision (CDPP) of 20 parts per million (PPM) on a magnitude 12 star for a 6.5-hour integration.**

TESS Mission
Transiting Exoplanet Survey Satellite

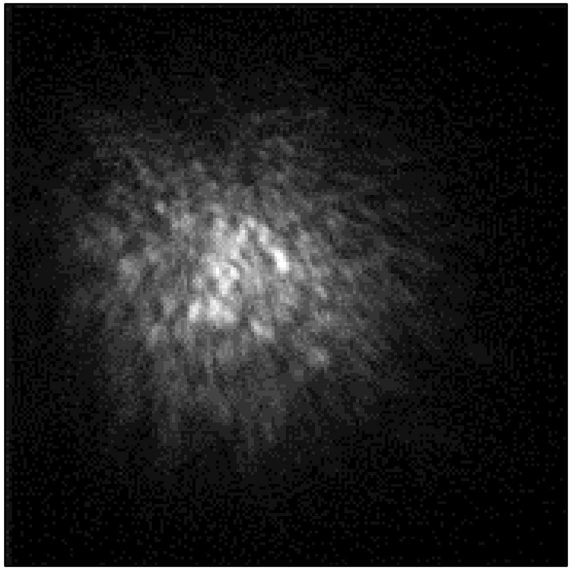
- **TESS's two-year all-sky survey will focus on nearby G, K, and M-type stars with apparent magnitude brighter than magnitude 12.**

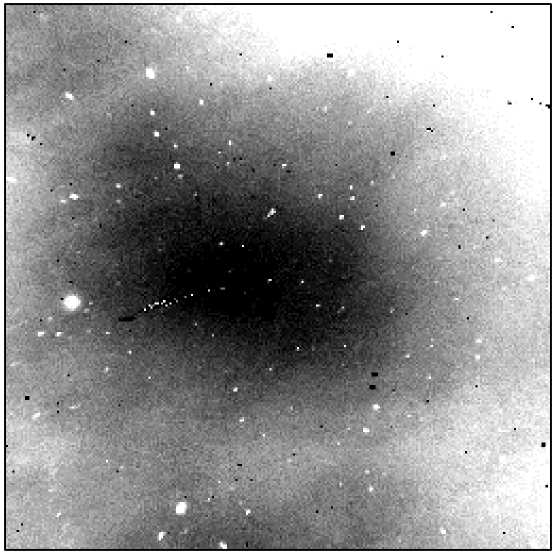
- **Approximately 500,000 stars will be studied, including the 1,000 closest red dwarfs across the whole sky, an area 400 times larger than that covered by the *Kepler* mission.**

- **The photometric precision for a 10th magnitude star is estimated to be about 200 ppm, so TESS will be sensitive to super-Earths around bright stars.**
-

Earth's Atmosphere Hurts

It absorbs, blurs, and even emits its own light.



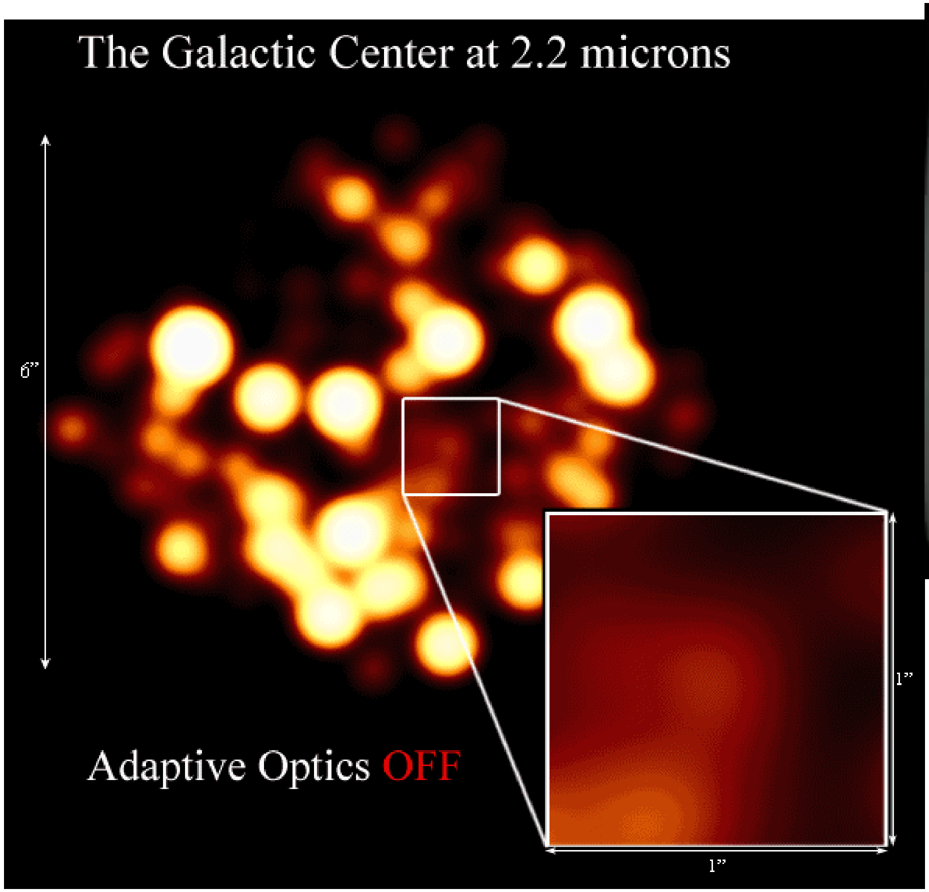


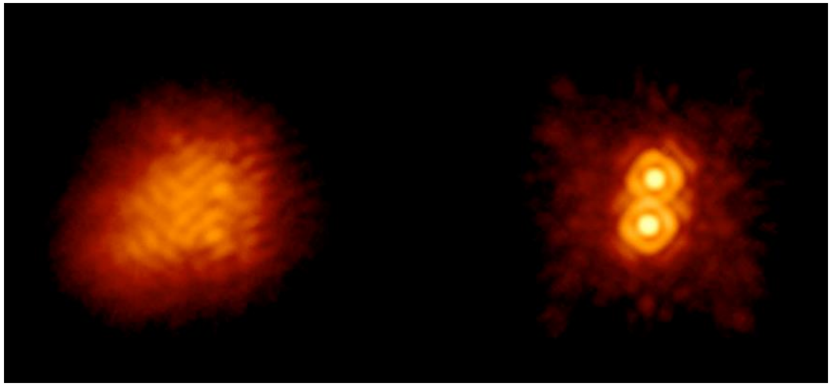
Turbulence blurs & twinkles starlight.

Atmosphere emits a background of IR light.

An infrared look through the atmosphere on a clear night

Adaptive Optics
“removing” atmospheric blurring





Direct Imaging

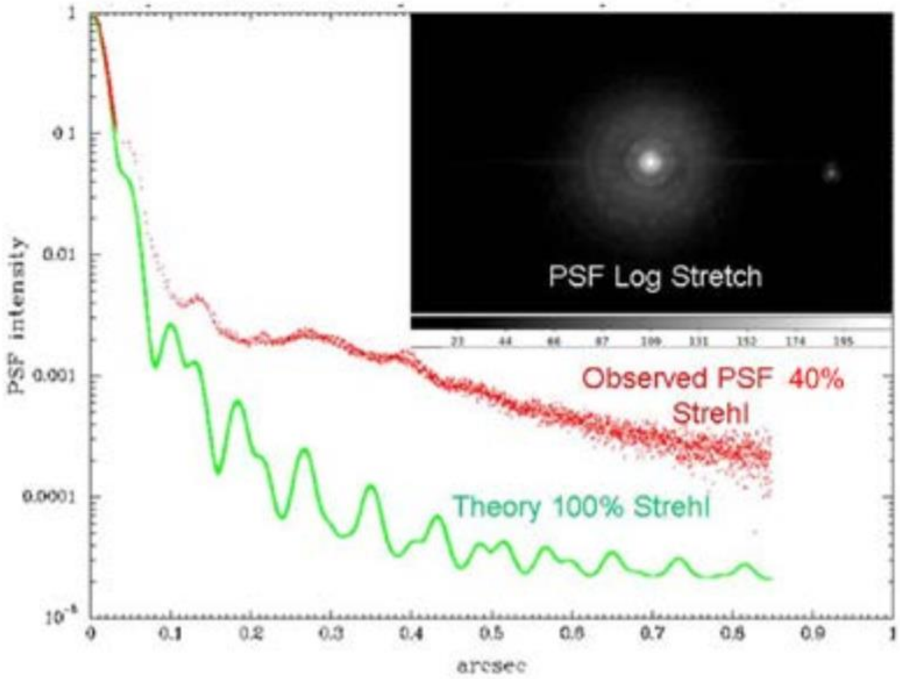


Fig 3: Deep PSF (40% Strehl) at 0.98 μm PSF (140 nm rms WFE with just 200 modes). Modified from Close et al. 2013. Note how the 35 mas resolution of the PSF allows for a raw contrast of 500 at just 0.1" separation. In this manner good inner working angles and contrasts can be achieved with visible AO even if the Strehls are lower than in the NIR.