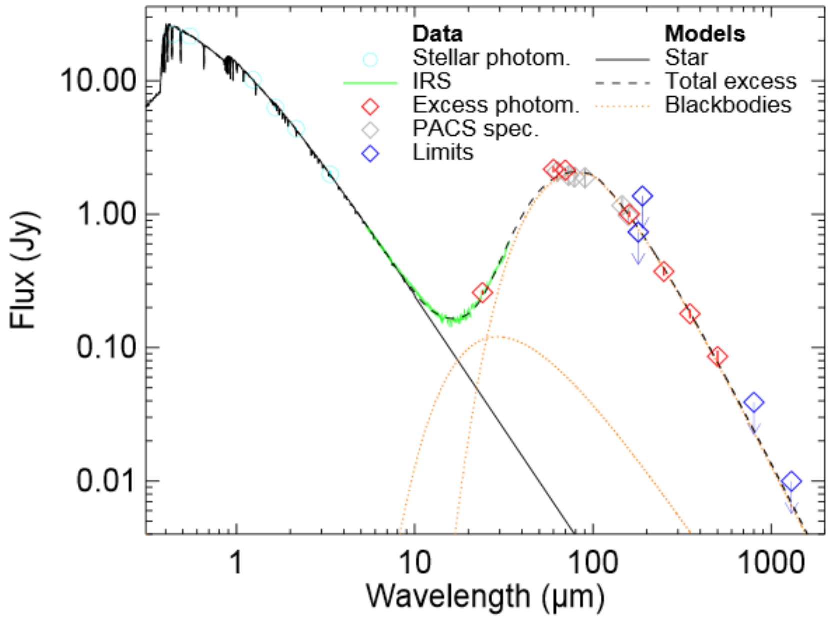
Comments: Homework #9

- **49 Ceti circumstellar dust**
  - **not opaque (used wrong equation?)**



- **consolidate before calculating**
  - **$(6000)^4 / (5000)^4$**
  - **$= (6000/5000)^4 = (6/5)^4$**
- **precision**
  - **5.69863296735 not necessary**
  - **implies a specific accuracy**
- **scientific notation**
  - **Use for all parts of your solutions**

Prove this Statement

"The energy needed to achieve low Earth orbit is roughly half of the total energy to reach anywhere in the Solar System."

Compare the kinetic energies.

$$\frac{1}{2} m v_{esc}^2 = \frac{1}{2} m(\sqrt{2} v_c)^2$$
$$\frac{1}{2} m v_{esc}^2 = m(v_c)^2$$

$$KE_{esc} = 2\ KE_C$$
$$\frac{1}{2} KE_{esc} = KE_C$$

Justifies the need for an orbital "waystation."

Vis-Viva Equation

speed at any point in an orbit

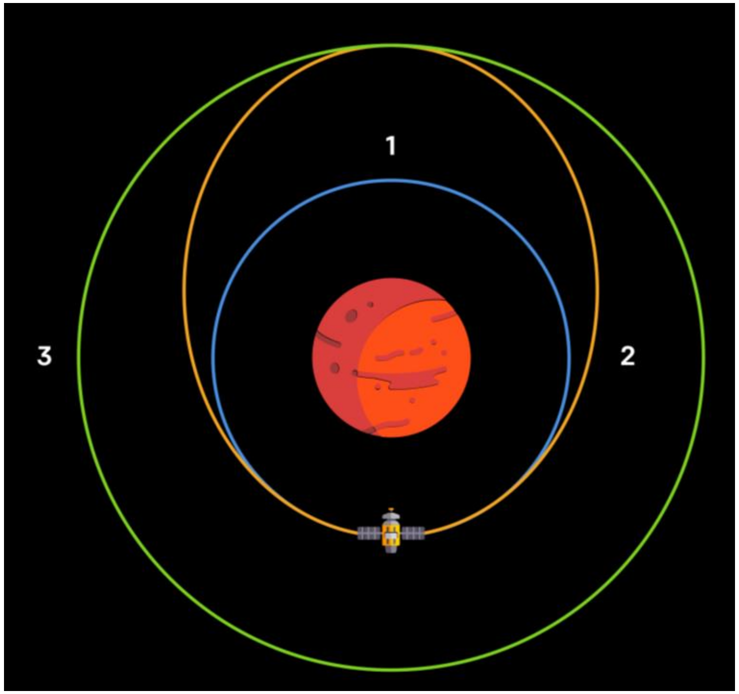
equation 3.67

$$v^2 = G(M+m)(\frac{2}{r} - \frac{1}{a})$$

What is v_{esc} from Earth to leave Solar System?

True or false:

"It's much more difficult to reach the Sun than it is to leave the Solar System altogether."



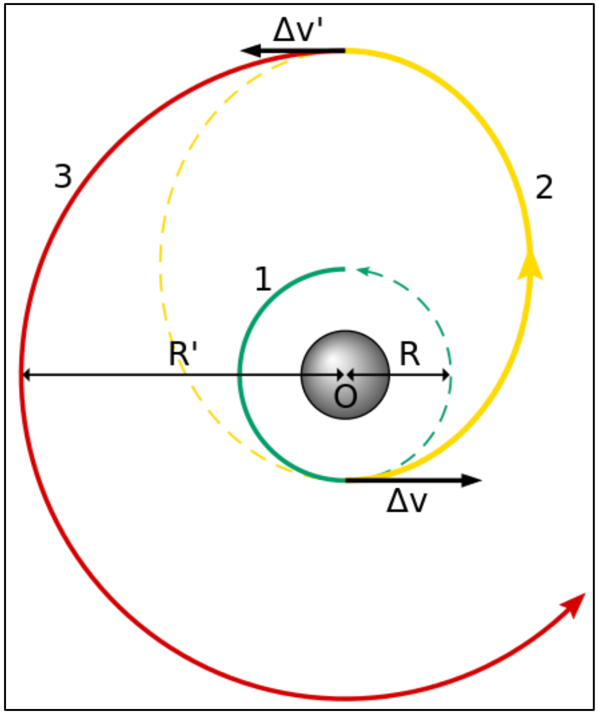
Hohmann transfer orbit

Hohmann Transfer Orbit

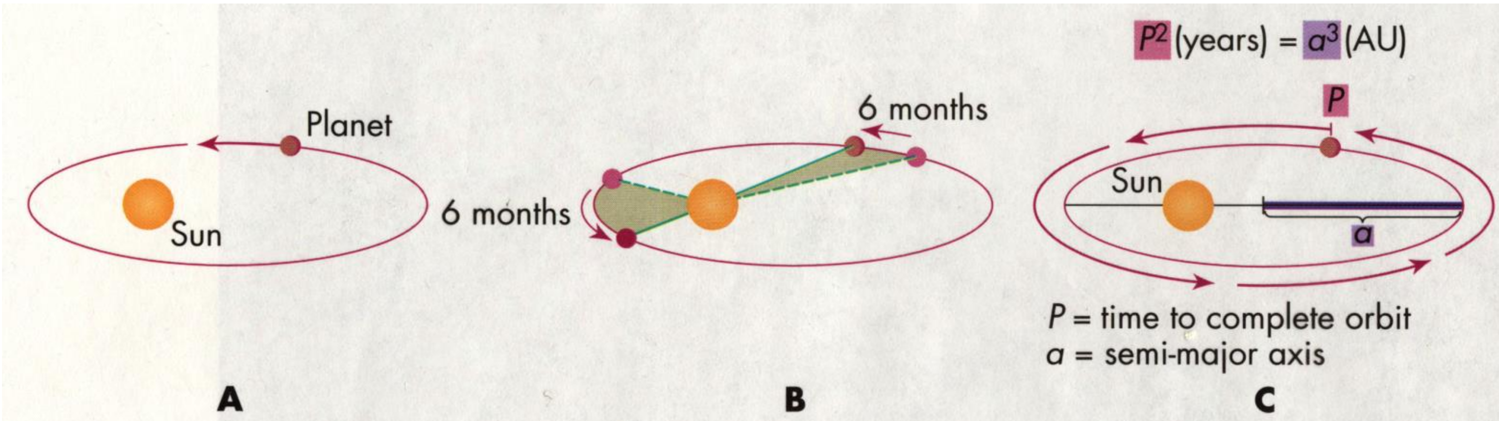
minimum energy usage => maximize payload

- **Shown: Earth-Mars**
  - **9 month travel time**
  - **"windows" open every 26 months**

- **Parker Solar Probe**
  - **ExCr: Calculate energy required to reach 0.04 AU vs to Mars**
  - **Using textbook, I got 60x more energy required!**



Newton's Law of Gravity Explains Kepler's THREE laws



Newton explained Kepler's Third Law ($P^2 = a^3$) using gravity. Newton found a more thorough mathematical form.

$$P^2 = \frac{4\pi a^3}{G(M_1+M_2)}$$

Knowing the orbit of an orbit (P and a), one can find the masses of the objects.

Dividing Equations

$$P^2 = \frac{4\pi A^3}{G(M_1+M_2)}$$

Consider two binary systems and distinguish between them with capital letters.

$$p^2 = \frac{4\pi a^3}{G(m_1+m_2)}$$

$$(P/p)^2 = (A/a)^3\,(m_1 + m_2) / (M_1 + M_2)$$

(continued)

$$(P/p)^2 = (A/a)^3 \, (m_1 + m_2) \, / \, (M_1 + M_2)$$

If the second system is the Earth-Sun, then p = 1 yr, a = 1 AU, and $(m_1 + m_2) = 1\ M_{sun}$.

This is equivalent to measuring P (yrs), A (AU), and M (solar masses):

$$P^2 = \frac{A^3}{(M_1+M_2)}$$

Kepler's Third Law

- **If you choose these units:**
  - **period (P) in years**
  - **semi-major axis (a) in AU**
  - **mass in "solar masses"**

- **Then, we can rewrite Kepler's Third Law:**
  - $P^2 = a^3 / M_{total}$

- **For a binary star:**
  - $P^2 = a^3 / (M_1 + M_2)$

Problem

- "Low-Earth orbit" definition:
  - altitude ~1/3 radius of Earth
- What is the period of a "low-Moon" orbit?
  - Use your knowledge of the Earth-Moon system.

Solution

- $P^2 = A^3/M$ for Earth
- $p^2 = a^3/m$ for Moon

- $A = 4/3\ R$ and $a = 4/3\ r$
  - $r/R = 0.2725$
  - $M/m = 81.3$

- $(p/P)^2 = (a/A)^3\ (M/m) = (r/R)^3\ (M/m) = (0.2725)^3\ (81.3)$

- $p/P = 1.28$
- $p = 115$ minutes (for $P = 90$ minutes) ~ 2 hours

Period of low circular orbit is independent of diameter

Interestingly the low circular orbit period about a body is independent of its size. It depends only on the density of the object.

$$T = 2\pi\sqrt{\frac{a^3}{Gm}}$$

Let's say that the low circular orbit altitude is 6% of the radius of a sphere with average density ρ. Then:

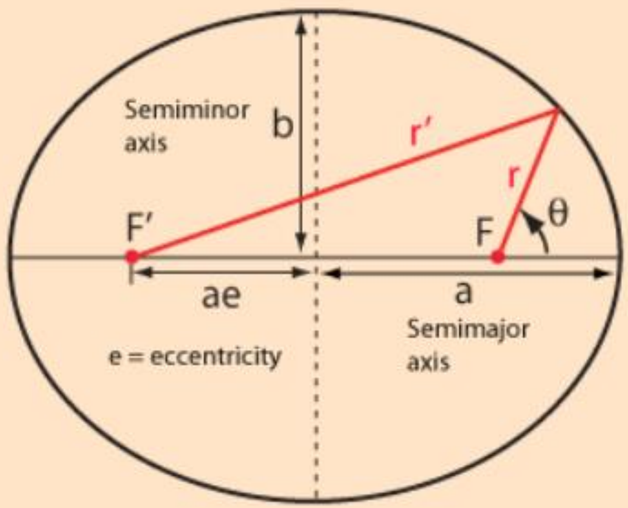
$$T = 2\pi\sqrt{\frac{3(1.06r)^3}{4\pi G\rho r^3}}$$

$$T \approx \frac{3.35}{\sqrt{G\rho}}$$

So low orbits around rocky bodies will be on the order of two hours, regardless of the size.

For Earth, $\rho = 5.51\,\mathrm{g/cm^3}$, which gives a period of 92 minutes. For the Moon with $\rho = 3.34\,\mathrm{g/cm^3}$, the period is 118 minutes. Mars' $\rho = 3.93\,\mathrm{g/cm^3}$, giving 109 minutes. Tiny Ceres at $\rho = 2.08\,\mathrm{g/cm^3}$ gives 150 minutes. (Dawn won't get that low.)

Problem
derive expressions for perihelion and aphelion



r:

$$r'^2 = r^2 \sin^2\theta + (2ae + r\cos\theta)^2$$

Using the trigonometric identity

$$\sin^2\theta + \cos^2\theta = 1$$

this reduces to

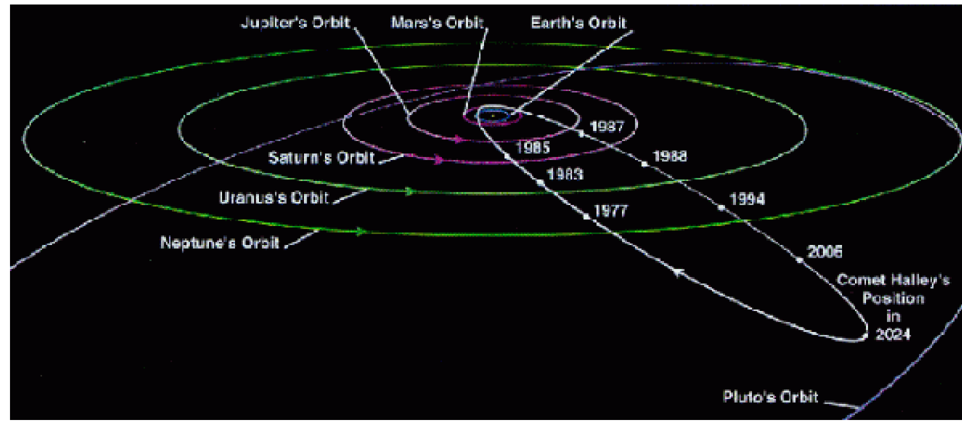
$$r'^2 = r^2 + 4ae(ae + r\cos\theta)$$

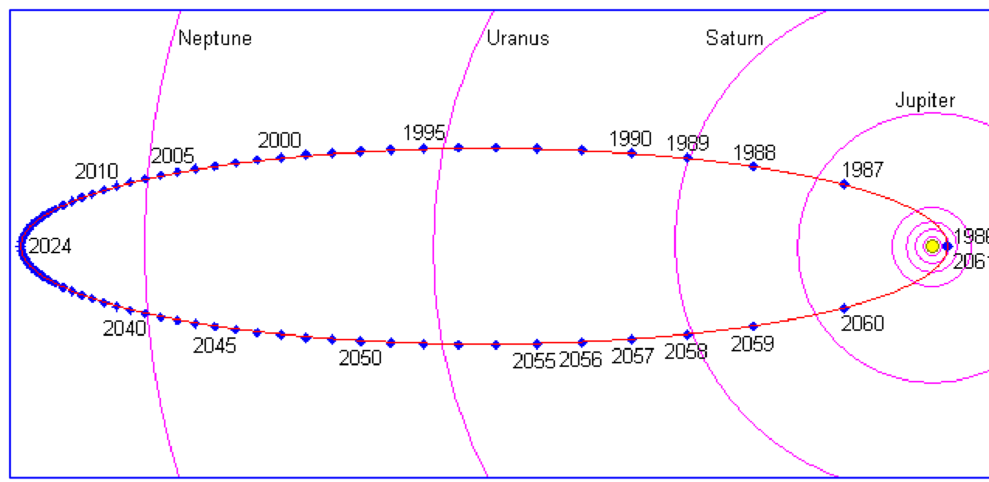
Using the equation for an ellipse, an expression for r can be obtained

$$r = \frac{a(1-e^2)}{1+e\cos\theta} \quad \textit{for} \quad 0 \le e < 1$$

Problem
Halley’s comet

Semi-major axis	17.834 AU
Eccentricity	0.96714
Orbital period	75.32 yr
Mean anomaly	38.38°
Inclination	162.26°
Longitude of ascending node	58.42°
Argument of perihelion	111.33°

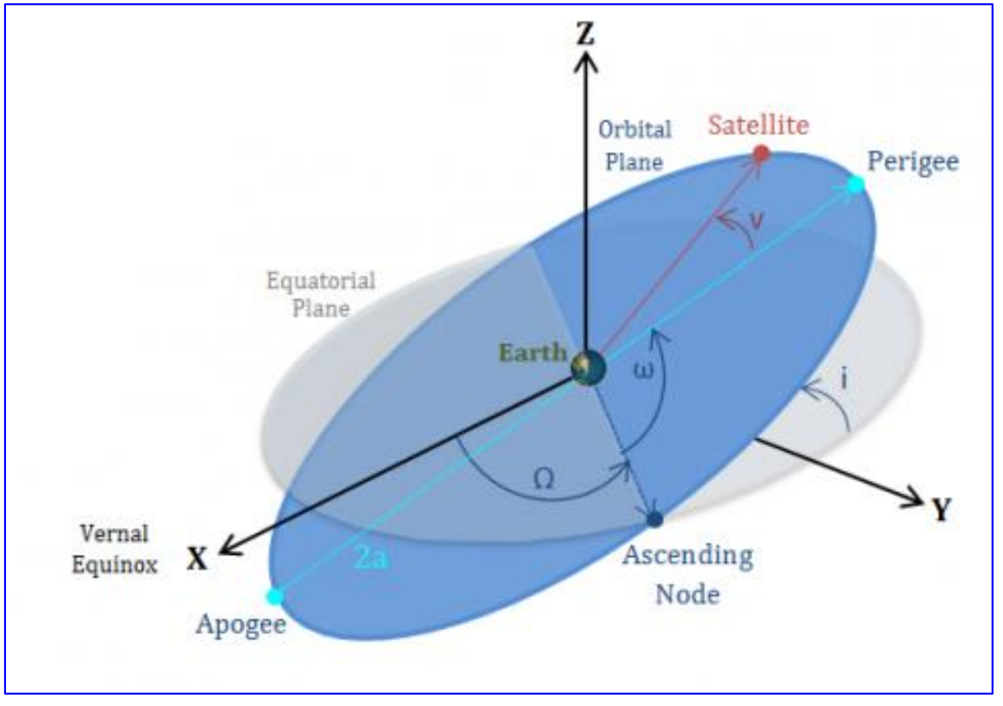




aphelion = ?

perihelion = ?

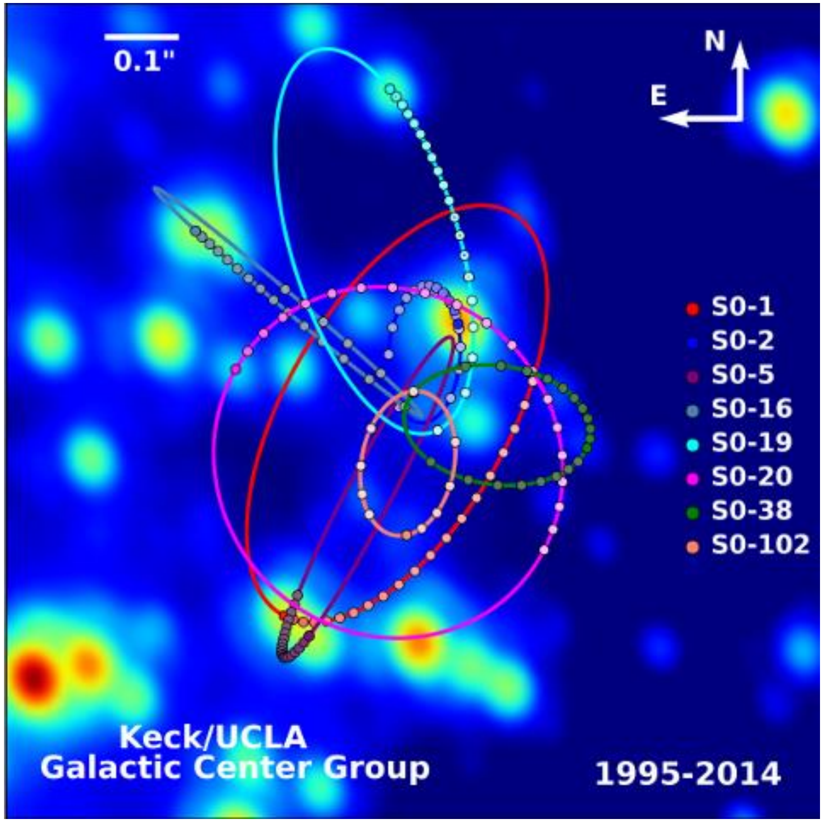
Orbital elements

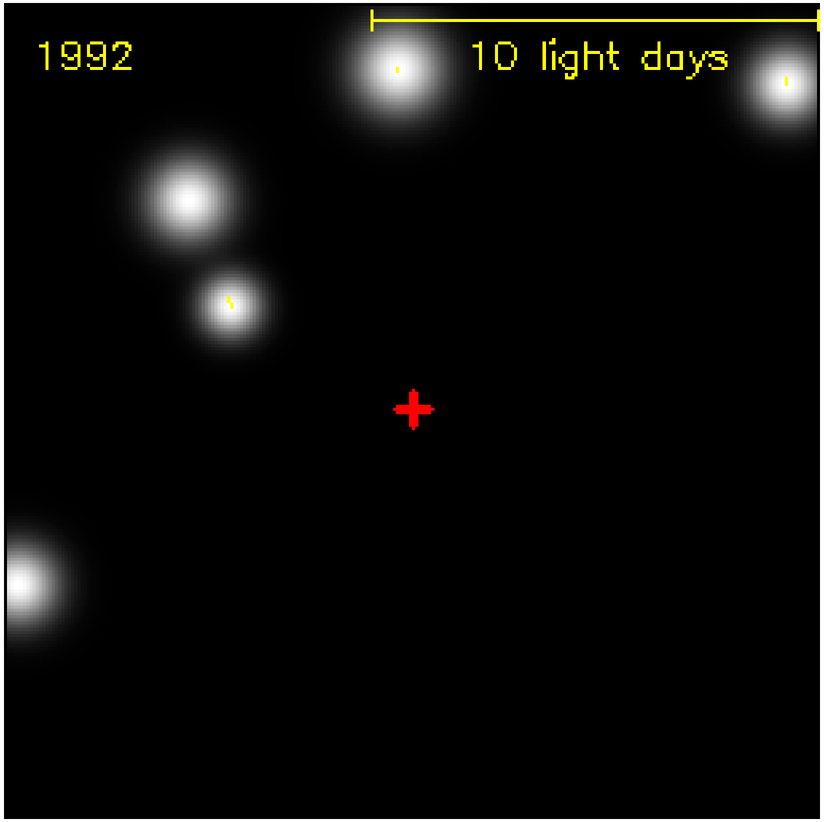


- size & shape of orbit
  - a, e
- orientation of orbital plane
  - i, Ω
- orientation of ellipse in the plane
  - ω
- position of object
  - ν

Stars Orbiting the Black Hole at the Center of Our Milky Way Galaxy

Do Kepler's Laws work there?





Virial Theorem

- **For any system of particles bound by an inverse-square force, the time-averaged kinetic energy <T> and the time-averaged potential energy <U> satisfy 2<T> + <U> = 0**

- **What assumptions?**
- **Why is U negative?**

- **Assumes a “bound” system**
  - **In equilibrium, neither expanding nor contracting**
  - **only gravitational forces**

- **Ignores rotation**
- **We will apply to globular clusters and the Coma cluster of galaxies**

Virial Theorem

$$2\langle T\rangle + \langle U\rangle = 0$$

- **$2T = Nmv^2 = Mv^2$**

- **m is individual mass**
- **N is number of objects with mass (m) and average speed (v)**

- **$U = -\alpha GM^2/R$**
- **α is a constant depending on how the mass is distributed**
- **R is the object's radius**

- **$Mv^2 = \alpha GM^2/R$**
- **$M = v^2R/\alpha G$**

Problem
globular star cluster (M71)

M71 Measurements

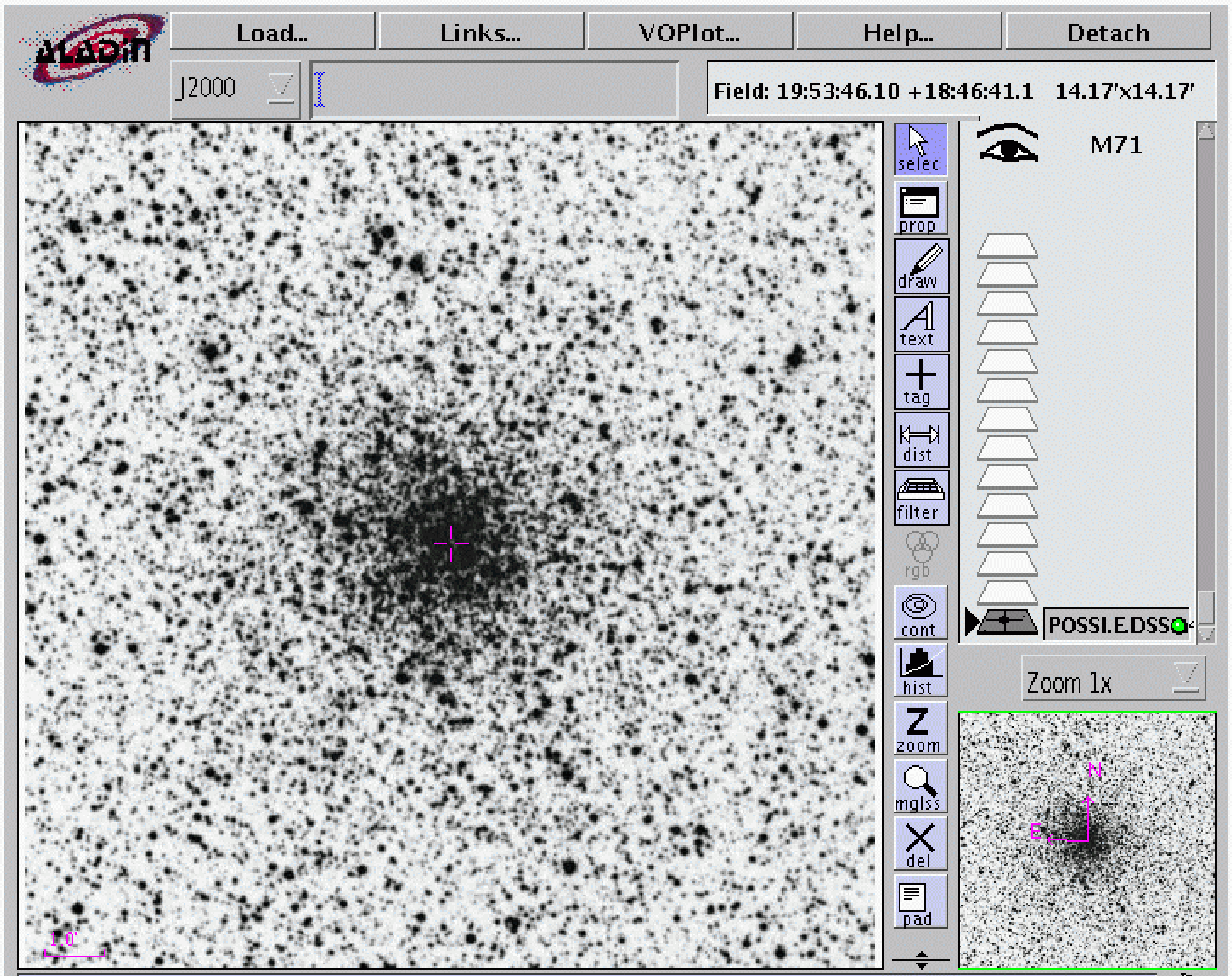
TABLE 1—(*continued*)

Star	M_{bol}^{e}	$(V-K)_0^{e}$	V	B-V	Disk No.	Julian Date	Exposure (min)	R	This Velocity	Other Velocity	Ref[b]	Notes
					NGC 6838 = M71 (Cudworth 1985a)							
A	...	...	10.70	0.53	2654-11	6222.984	2.5	10.7	-27.1 ± 0.7	-17 ± 7	PS83	0,f
					2656-15	6223.986	4	9.7	-27.0 ± 0.8			
B,V2	-3.13	4.66	12.08	1.87	2654-9	6222.980	7.2	5.5	-18.9 ± 1.3			95
					2656-13	6223.982	5.5	4.7	-19.2 ± 1.5			
A4	-2.35	3.55	12.20	1.69	2651-19	6221.902	6	10.3	-25.4 ± 0.7	-25	C80	94
A9	-1.38	3.37	12.94	1.57	2651-24	6221.931	12	11.2	-24.8 ± 0.7			86
I	...	...	12.42	1.57	2656-11	6223.976	8.2	10.4	-15.5 ± 0.7			93
S	-1.34	3.12	12.94	1.51	2651-22	6221.920	12	10.7	-23.7 ± 0.7			81
1-21	-1.08	2.84	13.02	1.49	2651-28	6221.956	15	12.7	-21.5 ± 0.6			95
1-29	-3.39	5.6	12.76	1.84	2651-17	6221.896	8	9.4	-23.6 ± 0.8			90
1-36	...	...	12.79	1.25	2651-30	6221.964	10	8.5	-22.2 ± 0.9	-24 ± 11	GN78	64
1-45	-2.18	3.60	12.36	1.76	2654-2	6222.948	12	9.2	-22.8 ± 0.8	-22	C80	94
					2654-4	6222.958	12	9.8	-23.2 ± 0.8	-29 ± 14	GN78	
										-22 ± 18	GN78	
1-46	-2.31	3.64	12.29	1.75	2656-9	6223.968	12	8.8	-23.2 ± 0.9	-26	C80	93
										-16 ± 10	GN78	
1-53	...	...	12.97	1.61	2652-1	6221.974	10	9.6	-25.1 ± 0.8			95
					2654-7	6222.972	15	9.5	-24.2 ± 0.8			
1-56	...	...	13.14	1.38	2656-7	6223.958	20	11.7	-20.9 ± 0.7			96
1-64	...	...	13.10	1.53	2653-26	6222.908	17	12.5	-17.2 ± 0.6	-24 ± 40	HS78	95
					2656-4	6223.943	6	7.7	-17.4 ± 1.0	-9 ± 40	HS78	
1-66	...	...	13.01	1.40	2653-28	6222.919	15	13.1	-20.0 ± 0.6			81
1-77	-1.89	3.57	12.65	1.73	2651-26	6221.941	12	9.7	-27.1 ± 0.8	-16 ± 19	GN78	73
1-113	-2.25	3.81	12.43	1.80	2653-24	6222.895	10	10.3	-21.5 ± 0.7	-29 ± 11	GN78	95
										-21 ± 18	GN78	

[a] In km s^{-1}.

Calculate the Virial Mass of Globular Star Cluster M71

distance = 3.8 kpc



Globular Cluster Properties

Table 1. Basic Facts about the Globular Clusters of the Galaxy

Number known	147
Median distance from Galactic Centre	9.3kpc
Median absolute V magnitude	-7.27
Median concentration	1.50
Median core relaxation time	3.39×10^8yr
Median relaxation time at the half-mass radius	1.17×10^9yr
Median core radius	1.32pc
Median half-mass radius	3.08pc
Median tidal radius	34.5pc
Median mass	$8.1 \times 10^4 M_\odot$
Median line-of-sight velocity dispersion	5.50km/s