HOMEWORK #13 (due start of class Feb 17)

LEARNING GOALS for this assignment:
1. To apply the Virial Theorem in astronomical settings.
2. To understand the physical origin of "tidal forces"
3. To understand the implications of tidal forces: Hill sphere, Roche Limit, orbital evolution, …

TO RECEIVE FULL CREDIT:
1. Staple multiple pages and identify yourself by Star Name (worth 5 points!).
2. You must **show how** you derived your answer by writing all the logical steps that led you to it. Follow the format of the "Homework Example" on our Web site.
3. All sentence responses must be typewritten and in complete sentences. You may handwrite any arithmetic. Use good English grammar.
4. **If you work more than three hours on this assignment, you should stop, record your work here, and contact Dr. McCarthy or Mr. Hammer (our Teaching Assistant) for help.**

Appling the Virial Theorem: Globular Cluster and Coma Cluster of Galaxies

Part I. Reading and Questions

In Ryden and Peterson's textbook, read Sections 4.1 through 4.3 (pp. 83-97) about the implications of the inverse-square law of the force of gravity.

a. Explain why the textbook's equation (3.84), shows a distance-cubed in the denominator.

b. Derive equation (4.42). Show how the approximation works mathematically.

Part II. Calculate the Virial Mass of Globular Star Cluster (M71)

Complete the project we began during Friday's class: Calculate the virial mass of M71 using the information provided which is reproduced below.

$$\text{virial mass} = v^2R/\alpha G, \text{ where } \ldots$$

R is the cluster's radius;
v is the "velocity dispersion" of the stars within the cluster;
$\alpha = 0.6$ (Homework #12); a constant depending on the distribution of mass in the cluster;
$G = 6.67 \times 10^{-11}\ m^3\ kg^{-1} sec^{-2}$.

You can determine the cluster's radius (R) from the following image of M71. This image has an angular field-of-view of 14.17 arcminutes on a side. Estimate the cluster's angular diameter using the image. Then use the small-angle equation to calculate the cluster's radius. The distance to M71 is 3.8 kpc.
[NOTE: The cluster is viewed in the direction of our Milky Way, so most of the stars you see are either foreground or background stars, not part of the cluster. Stars in the cluster are gravitationally bound.]

The average speed of the stars is -22.34 +/- 3.29 km/sec. The average value of -22.34 km/sec is the cluster's so-called "radial velocity" and indicates that the entire cluster is travelling towards us. The "velocity dispersion (v)" of 3.29 km/sec is a measure of the internal motion of the stars within the cluster.

Other constants that might be useful:
$M_{Sun} = 2 \times 10^{30}$ kg

Part III. Calculate the Virial Mass of the Coma Cluster of Galaxies

In 1933, Fritz Zwicky calculated the mass of the Coma cluster, a galaxy cluster at a distance of ~100 Mpc. He computed this mass in two ways: First, by simply counting the number of galaxies and estimating their masses from their sizes and luminosities and, second, by using the Virial Theorem.

Based on the following facts, calculate the virial mass of the Coma cluster and compare that value to the luminous mass of ~10^{12} M_{Sun} (i.e., ~1000 visible galaxies, each with ~10^{9} M_{Sun}).

R = 2 x 10^{6} light-years (613 kpc)
Velocity dispersion = ~1000 km/sec

FOR PART II (M71):