HOMEWORK #12 (due start of class Feb 14)

LEARNING GOALS for this assignment:

1. To understand the physical origin and implications of the Virial Theorem.
2. To apply this understanding in practical situations in modern astronomy.

TO RECEIVE FULL CREDIT:

1. Staple multiple pages and identify yourself by Star Name (worth 5 points!).
2. You must **show how** you derived your answer by writing all the logical steps that led you to it. Follow the format of the "Homework Example" on our Web site.
3. All sentence responses must be typewritten and in complete sentences. You may handwrite any arithmetic. Use good English grammar.
4. **If you work more than three hours on this assignment, you should stop, record your work here, and contact Dr. McCarthy or Mr. Hammer (our Teaching Assistant) for help.**

The Virial Theorem

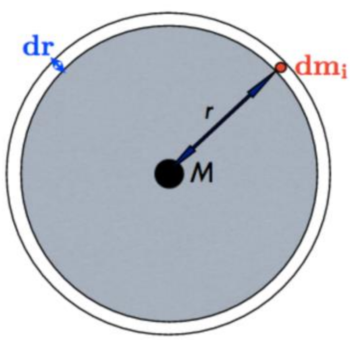
Part I. Reading

In Ryden and Peterson's textbook, re-read Section 3.4 *"The Virial Theorem."*

What are some of the assumptions inherent in the derivation of the Virial Theorem as derived in the reading?

Part II. Potential Energy of a Collapsing Sphere

In this problem, you will derive the gravitational potential energy, U_g, of a uniform density sphere (ρ = constant). Consider the geometry of a test mass, dm_i, within a thin spherical shell with differential radius dr:



The test mass has a gravitational potential energy of:

$$dU_{g,i} = -G\frac{M_r\, dm_i}{r}$$

where M_r is the total mass interior to radius r. The gravitational potential energy of the shell is then found by substituting dm = volume of shell * density = $4\pi r^2\rho dr$ to obtain

$$dU_g = -G\frac{M_r\, 4\pi r^2 \rho}{r}\, dr$$

a. To calculate the total gravitational potential energy of a sphere U_g, integrate dU_g over all mass shells from the center to radius R. Write down this integral and then pull everything that doesn't depend on r outside the integral. [NOTE: Capital R is the total radius of the sphere.]

b. How is M_r related to r and ρ? Assume ρ is constant. Substitute for M_r and evaluate the integral. Your answer should only contain numbers, G, with ρ and R as variables.

c. Now substitute for ρ to convert your answer to only have numbers, G, and M (total mass) and R as variables.

d. As the Sun contracted to its present radius, the virial theorem states that half of the total gravitational potential energy ($U_g/2$) was converted into kinetic energy and half radiated away in the form of light. At the present luminosity of the Sun ($L_{Sun} = 3.8 \times 10^{26}$ W), how long would it take the Sun to radiate that energy away (answer in years)? Can this account for the 4.6 Gyr lifetime of the Sun?