4) Understanding the Problem

- Calculate the temperatures of the two debris disks surrounding 49 Ceti using a graph of their blackbody curves
- Determine the distances of this material from 49 Ceti in AU, using knowledge of equillibrium temperature
- Wien's Law can be used to calculate temperatures from blackbody curve data

Convert nm→km→AU?

  - $\lambda_{max} = \frac{3 \cdot 10^6 \text{ nm}}{T} \rightarrow T = \frac{3 \cdot 10^6 \text{ nm}}{\lambda_{max}} = \frac{3\,\mu m}{\lambda_{max}}$

- Equillibrium Temperature formula: $T^4 = \frac{(1-A) \cdot \left(\frac{L_{star}}{4\pi D^2}\right)}{4\sigma}$
- Distance formula: $D^2 = (1-A) \cdot \left(\frac{L_{star}}{16\pi\sigma T^4}\right)$
  - A = albedo
  - σ = Stefan-Boltzmann constant
  - L_{star} = Luminosity of 49 Ceti

Devising a Plan

- Determine the wavelengths at which each blackbody curve (debris disks) is at its respective maximum luminosity
  - Substitute these values into Wien's Law equation for the λ_{max} variable and solve the equation for temperature, T,
- Estimate values for the albedo of the two debris disks
- Use the values for albedo and temperature of the debris disks to determine their distance from 49 Ceti

Carrying Out the Plan

① $T_{Disk A} = \frac{3 \cdot 10^6 \text{ nm}}{(8 \cdot 10^4) \text{ nm}} = \boxed{37.5 \text{ K}}$

② $T_{Disk B} = \frac{3 \cdot 10^6 \text{ nm}}{3 \cdot 10^4 \text{ nm}} = \boxed{100 \text{ K}}$

③ Albedo estimates

- Disk A = 0.50
- Disk B = 0.30

④ $D^2_{Disk A} = (1-0.5) \cdot \left(\frac{25 \cdot 3.8 \cdot 10^{26} \text{ W}}{16\pi \cdot 5.67 \cdot 10^{-8} \text{ W/m}^2/\text{K}^4 \cdot (37.5 \text{ K})^4}\right)$

⑤ $D_{Disk A} = \sqrt{0.5 \cdot \frac{9.5 \cdot 10^{27} \text{ W}}{16\pi \cdot 5.67 \cdot 10^{-8} \text{ W/m}^2 \cdot 37.5^4}}$

⑥ $D_{Disk A} = \frac{3 \cdot 10^{13} \text{ m}}{10^3 \text{ m}} = \frac{3 \cdot 10^{10} \text{ km}}{1.496 \cdot 10^8 \text{ km}} = \boxed{200.5 \text{ AU}}$

⑦ $D^2_{Disk B} = (1-0.3) \cdot \sqrt{\frac{25 \cdot 3.8 \cdot 10^{26} \text{ W}}{16\pi \cdot 5.67 \cdot 10^{-8} \text{ W/m}^2/\text{K}^4 \cdot (100 \text{ K})^4}}$

4 cont.) (8) $D_{Disk\,B} = \sqrt{0.7 \cdot \frac{9.5 \cdot 10^{27}\,W}{16\pi \cdot 5.67 \cdot 10^{-8} W/m^2 \cdot (100)^4}}$

(9) $D_{Disk\,B} = \frac{4.8 \cdot 10^{12}\,m}{10^3\,m} = \frac{4.8 \cdot 10^{9}\,km}{1.496 \cdot 10^{8}\,km} = \boxed{32\,AU}$

Looking Back

My results for the distance of each debris disk from 49 Ceti are logical. Because Disk A has a lesser temperature than Disk B, I estimated its albedo value to be less than that of Disk B since having a lesser temperature indicates its absorbing efficiency (albedo) is less than the absorbing efficiency of Disk B.

Given this and the temperatures of the disks, it makes sense for Disk A to be further away from 49 Ceti than Disk B because it does not absorb as much light and, therefore energy, as Disk B and thus is cooler.

5) Understanding the Problem

- Determine how many photons the human eye can sense from a star with an apparent magnitude of 6
- Estimates about the area of the eye and length of time the eye is exposed to light are needed to determine number of photons we can see
- The Sun's apparent visual magnitude is -26.9
- Radiation from the Sun received at Earth's surface which is visible is $10^{-2} \frac{J}{sec/cm^2}$ $= 10^{-2}\ W/cm^2$
- A visual photon of light carries energy $E = hc/\lambda$

  $h = 6.63 \cdot 10^{-34}\ m^2 kg/s$

  $c = 3 \cdot 10^8\ m/s$
- The formula for comparing apparent magnitudes is $m_1 - m_2 = 2.5 \log\left(\frac{B_2}{B_1}\right)$

Devising a Plan

- Solve for the brightness of the Sun (B_2) using values of Sun from Homework #6
- Substitite the known values of m_1 (eye's max apparent magnitude), m_2 (Sun's apparent magnitude), and B_2 (Sun's brightness) into the difference of apparent magnitude's equation
  - Solve for B_1 to indicate the energy, in W/m^2, the eye receives from the star with apparent magnitude 6
    - Convert this value to W/cm^2 to correspond to the given values of visual solar radiation
- Solve for the visible radiation on Earth, in W/cm^2, Earth receives from the star of 6 apparent magnitude
- Estimate area, in cm^2, exposure time, in seconds, and efficiency in W of the human eye to calculate the number of photons it can see from the star in question

Carrying Out the Plan

(1) $\text{brightness}_{Sun} = \frac{\text{luminosity}}{4\pi(\text{distance})^2} = \frac{3.8 \cdot 10^{26}\ W}{4\pi(1.496 \cdot 10^8\ km)^2} = \frac{1.35 \cdot 10^9\ W/km^2}{10^6\ m^2} = 1.35 \cdot 10^3\ W/m^2 = B_2$

(2) $m_1 - m_2 = 2.5 \log\left(\frac{B_2}{B_1}\right)$

$6 - (-26.9) = 2.5 \log\left(\frac{1.35 \cdot 10^3\ W/m^2}{B_1}\right)$

(3) $\frac{32.9}{2.5} = \log\left(\frac{1.35 \cdot 10^3\ W/m^2}{B_1}\right)$

5 cont.) Carrying Out the Plan (cont.)

(4) $10^{(32.9/2.5)} = \frac{1.35 \cdot 10^3\ W/m^2}{B_1}$

(5) $B_1 = \frac{1.35 \cdot 10^3\ W/m^2}{10^{(32.9/2.5)}} = 9.34 \cdot 10^{-11}\ W/m^2 = \frac{9.34 \cdot 10^{-11}\ W}{\cancel{m^2}} \cdot \frac{1\cancel{m^2}}{10^4\ cm^2} = 9.34 \cdot 10^{-15}\ W/cm^2$

(6) Radiation from star with 6 apparent magnitude in visible spectrum on Earth

i. $\frac{B_{2(Sun)}}{R_{Visual(Sun)}} = \frac{B_{1(Star)}}{R_{Visual(Star)}} \rightarrow R_{Visual(Star)} = \frac{B_1 \cdot R_{Visual(Sun)}}{B_2}$

ii. $R_{Visual(Star)} = \frac{9.34 \cdot 10^{-15}\ W/cm^2 \cdot .01\ W/cm^2}{1.35 \cdot 10^3\ W/m^2 \cdot 10^{-4}\ W/cm^2} = \frac{9.34 \cdot 10^{-17}\ W^2/cm^4}{1.35 \cdot 10^{-1}\ W^2/cm^2} = 6.92 \cdot 10^{-16}\ W/cm^2$

(7) Estimates for characteristics of the human eye

Radius at night: $1\ mm = 10^{-1}\ cm$

Area at night: $\pi \cdot (10^{-1}\ cm)^2 = \pi \cdot 10^{-2}\ cm^2$

Exposure time: 5 seconds

Efficiency: 20%

(8) $R_{Visual(Star)} \cdot Area_{eye} = 6.92 \cdot 10^{-16}\ W/cm^2 \cdot \pi \cdot 10^{-2}\ cm^2 = 2.17 \cdot 10^{-17}\ W$

(9) $2.17 \cdot 10^{-17}\ W \cdot 5$ seconds

$2.17 \cdot 10^{-17}\ \frac{Joules}{seconds} \cdot 5\ seconds = 1.085 \cdot 10^{-16}\ J$

(10) $1.085 \cdot 10^{-16}\ J \cdot 0.20 = 2.17 \cdot 10^{-17}\ J \cdot 10^{-4}\ cm^2 = 2.17 \cdot 10^{-21}\ J$

(11) $E = \frac{hc}{\lambda} \rightarrow 2.17 \cdot 10^{-21}\ J = \frac{6.63 \cdot 10^{-34}\ \frac{m^2 kg}{s} \cdot 3 \cdot 10^8\ m/s}{\lambda}$

(12) $\lambda = \frac{2 \cdot 10^{-25}\ \frac{m^3 kg}{s^2}}{2.17 \cdot 10^{-21}\ J} = 9.22 \cdot 10^{-5}\ m$

(13) $\frac{\lambda}{E} = \#\ of\ photons \rightarrow \frac{9.22 \cdot 10^{-5}\ m}{2.17 \cdot 10^{-21}\ J} = \boxed{4.25 \cdot 10^{16}\ photons}$

5 cont.) Looking Back

This answer makes sense because the eye needs a large number of photons to create images so this value is reasonable, especially given the faintness of the star at an apparent magnitude of 6. As for the accuracy of this result, I estimated values for each feature of the eye which was relevant in these calculations. Therefore, this result may not be accurate if my approximations for the human eye were inaccurate.